Kinematics, Dynamics, and Vibrations FE Review Session

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Example 1 A 10 g ball is released vertically from a height of 10 m. The ball strikes a horizontal surface and bounces back. The coefficient of restitution between the surface and the ball is 0.75. The height that the ball will reach after bouncing is most nearly

$$KE = PE$$

$$e = \frac{v_r}{v_0}$$

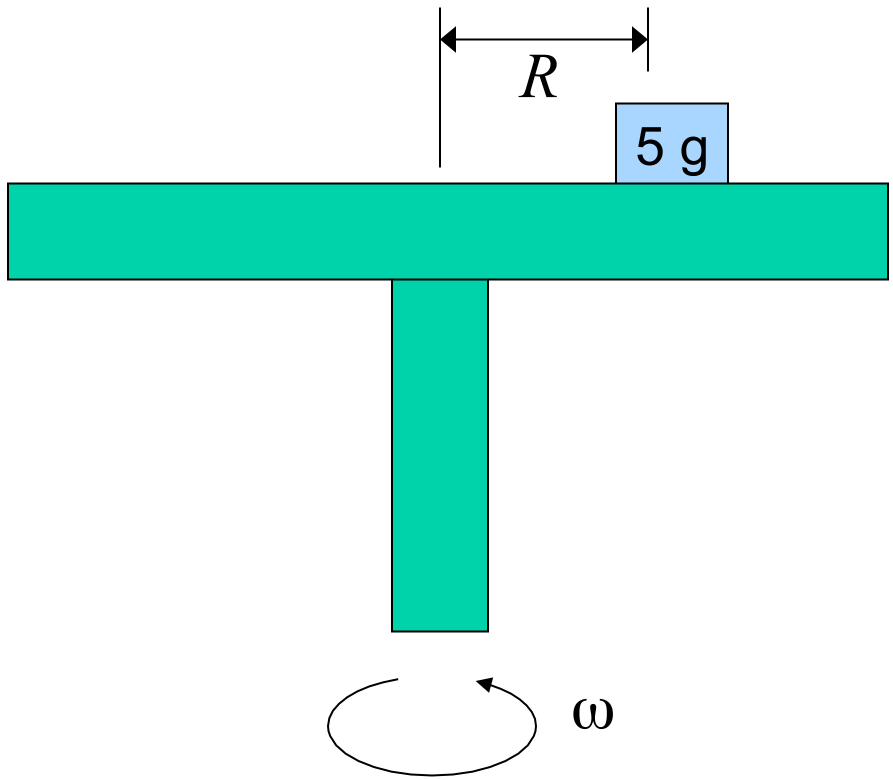
$$\frac{1}{2}mv_0^2 = mgh_0$$

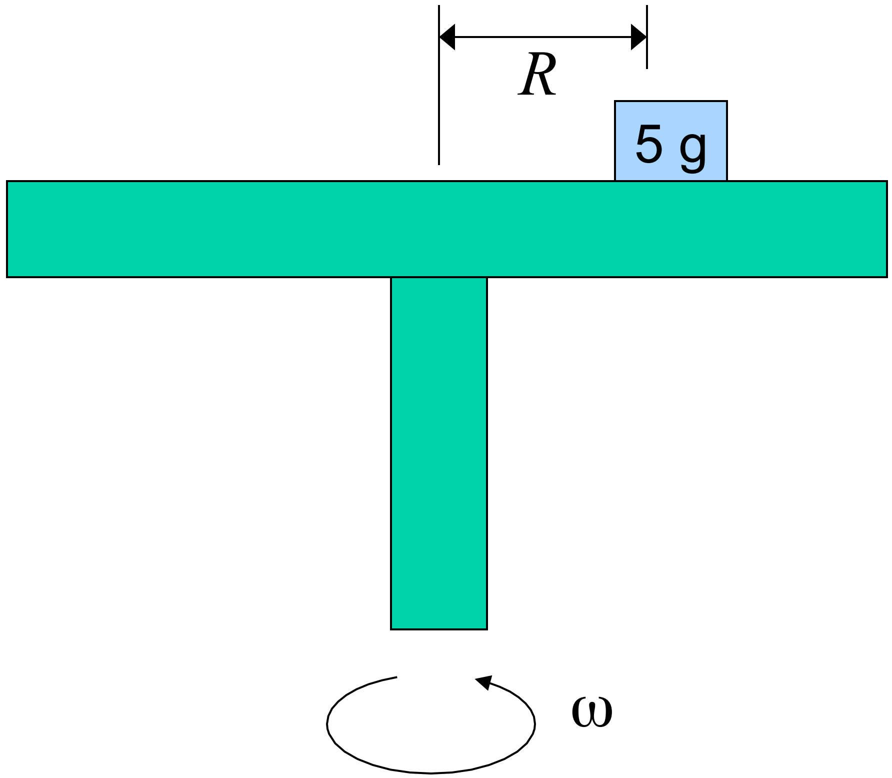
$$\frac{1}{2}mv_r^2 = mgh_r$$

$$v_0 = \sqrt{mgh_0}$$

$$h_r = 5.6\,m$$

Example 2 A 5 g mass is to be placed on a 50 cm diameter horizontal table that is rotating at 50 rpm. The mass must not slide away from its position. The coefficient of friction between the mass and the table is 0.2. What is the maximum distance that the mass can be placed from the axis of rotation?





$$\sum F = ma_r$$

$$-\mu N = -m\omega^2 R$$

$$R = \frac{\mu g}{\omega^2} = .072\,m$$

Example 3 A truck of 4000 kg mass is traveling on a horizontal road at a speed of 95 km/h. At an instant of time its brakes are applied, locking the wheels. The dynamic coefficient of friction between the wheels and the road is 0.42. The stopping distance of the truck is most nearly

(A) 55 m

(B) 70 m

(C) 85 m

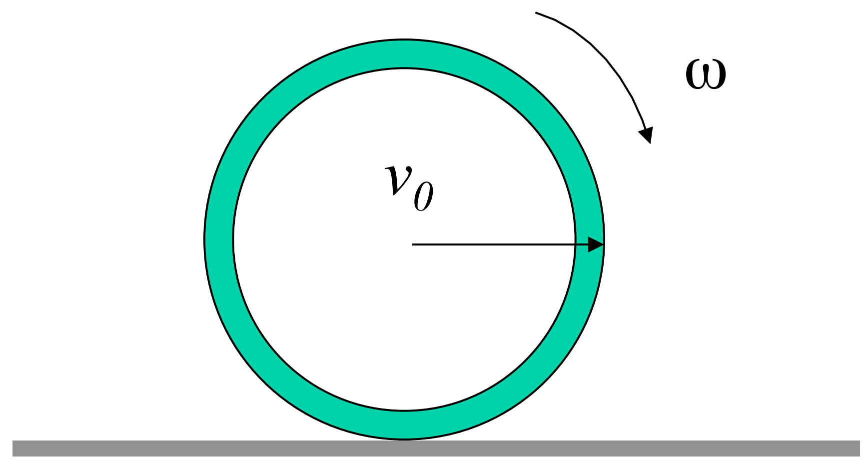
(D) 100 m

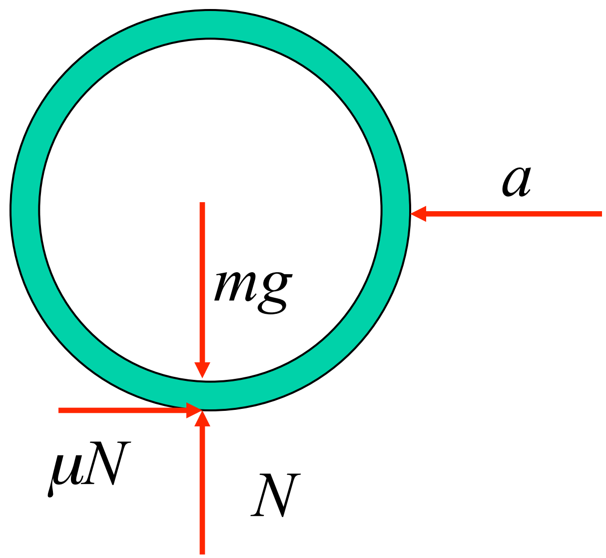
$$KE\Big|_1 = KE\Big|_2 + W$$

$$\frac{1}{2}mv^2 = \mu_k N \Delta x$$

$$\Delta x = \frac{1}{2}mv^2 \frac{1}{\mu_k N} = 84.5\ m$$

Example 4 A translating and rotating ring of mass 1 kg, angular speed of 500 rpm, and translational speed of 1 m/s is placed on a horizontal surface. The coefficient of friction between the ring and the surface is 0.35. For an outside radius of 3 cm, the time at which skidding stops and rolling begins is most nearly





$$\sum F_x = ma \qquad \sum F_y = 0$$

$$\mu N = ma \qquad N = mg$$

$$\sum M = I\alpha$$

$$\mu NR = mR^2\alpha$$

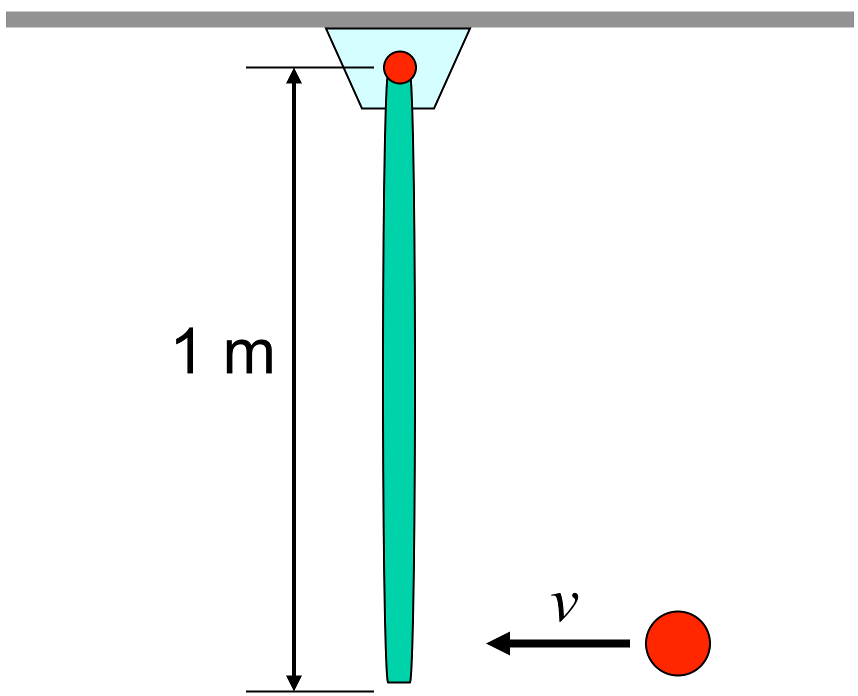
$$v = v_o + at$$

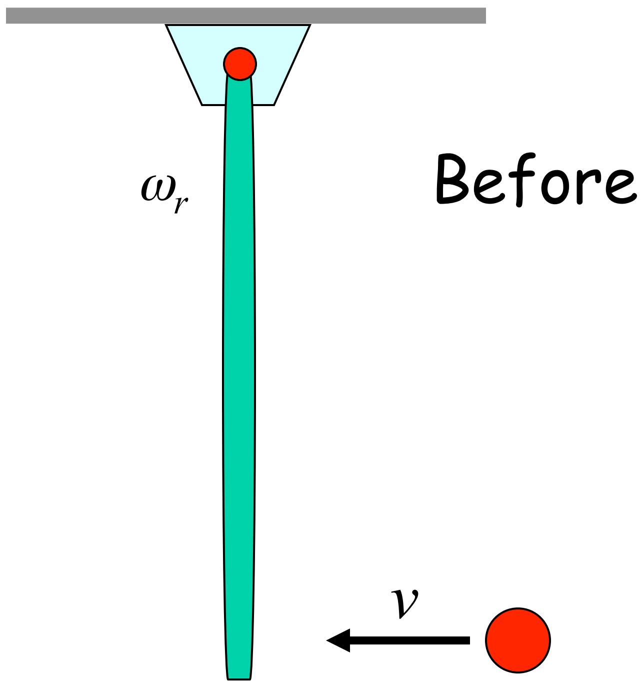
$$\omega = \omega_0 - \alpha t$$

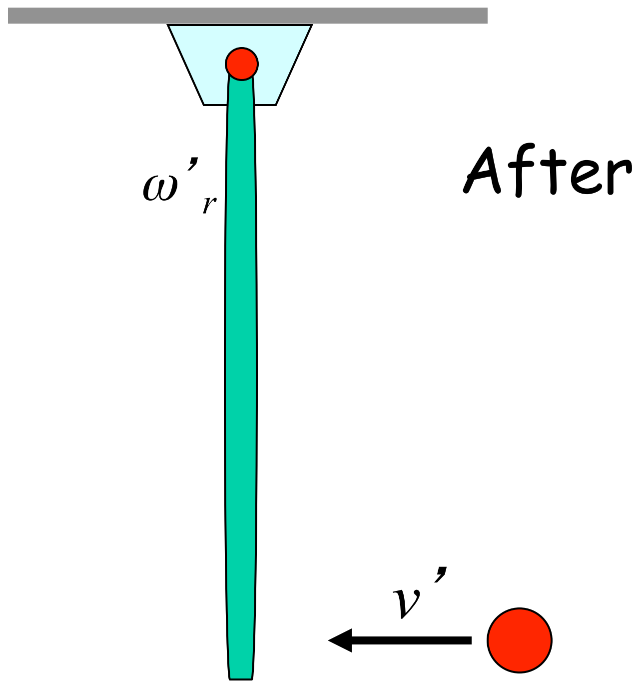
Skidding Stops when

$$v = R\omega$$

Example 5 A stationary uniform rod of length 1 m is struck at its tip by a 3 kg rigid ball moving horizontally with velocity of 8 m/s as shown. The mass of the rod is 7 kg, and the coefficient of restitution between the rod and the ball is 0.75. The velocity of the ball after impact is most nearly







$$mvL = mv'L + \frac{1}{3}m_r L^2 \omega'$$

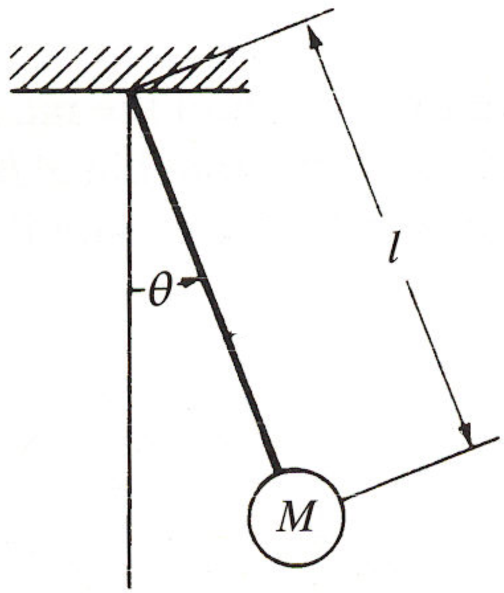
$$e = \frac{v'_r - v'}{v - v_r}$$

$$v' = 1.88\ m/s$$

Example 6 The D’ Alembert force is

(A) Force of gravity in France.
(B) Force due to inertia.
(C) Resisting force due to static friction
(D) Atomic force discovered by D’ Alembert.

Example 7 A 5 kg pendulum is swung on a 7 m long massless cord from rest at 5° from center. The time required for the pendulum to return to rest is most nearly



$$\omega_n = \sqrt{\frac{g}{L}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$f = 0.1885\,Hz$$

$$T = \frac{1}{f}$$

Example 8 A 2 kg mass is suspended by a linear undamped spring with a spring constant 3.2 kN/m. The mass is given an initial velocity of 10 m/s from the equilibrium position.

Example 8.1 Calculate the natural frequency of the mass.

$$\omega_n = \sqrt{\frac{k}{m}} = 40 \ rad/s$$

Example 8.2 How long does it take for the mass to complete one complete cycle.

$$f = \frac{1}{2\pi}\omega_n$$

$$T = \frac{1}{f} = 0.156\ s$$

Example 8.3 What is the maximum deflection of the spring from the equilibrium position?

$$x(t) = x_0 \cos(\omega_n t) + \frac{v_0}{\omega_n} \sin(\omega_n t)$$

Example 9 A 115 kg motor turns at 1800 rpm, and it is mounted on a pad having a stiffness of 500 kN/m. Due to an unbalanced condition, a periodic force of 85 N is applied in a vertical direction, once per revolution. Neglecting damping and horizontal movement, the amplitude of vibration is

$$\omega_n = \sqrt{\frac{k}{m}} = 66\ rad/s$$

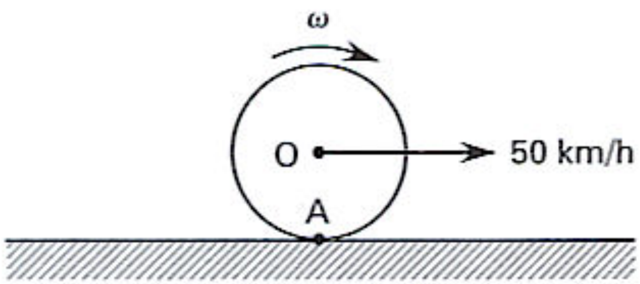
$$\omega_f = 188.5\ rad/s$$

$$\delta_{pst} = \frac{F_o}{k} = 1.7E-4\,m$$

$$\beta = \left| \frac{1}{1-\left(\frac{\omega_f}{\omega_n}\right)^2 + 2C\left(\frac{\omega_f}{\omega_n}\right)^2} \right| = 0.14\,m$$

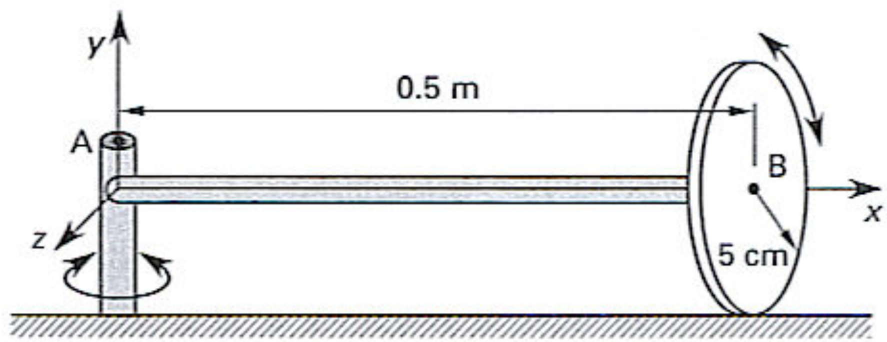
$$D = \beta\delta_{pst} = 0.024\,mm$$

Example 10 A uniform disk of 10 kg mass and 0.5 m diameter rolls without slipping on a flat horizontal surface, as shown. When its horizontal velocity is 50 km/h, the total kinetic energy of the disk is most nearly



$$KE = \frac{1}{2} m v_o^2 + \frac{1}{2} I_o \omega^2$$

Example 11 A homogeneous disk of 5 cm radius and 10 kg mass rotates on an axle AB of length 0.5 m and rotates about a fixed point A. The disk is constrained to roll on a horizontal floor. Given an angular velocity of 30 rad/s in the x direction and -3 rad/s in the y direction, the kinetic energy of the disk is



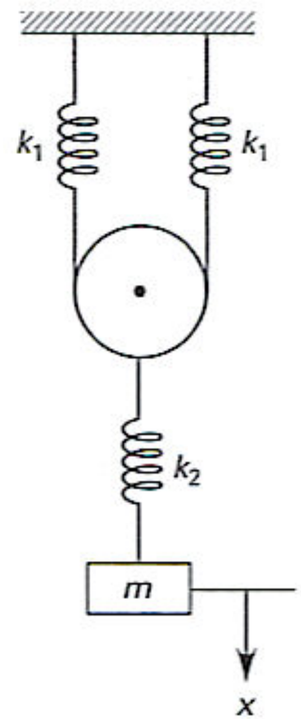
$$KE = \frac{1}{2} I_x \omega_x^2 + \frac{1}{2} I_y \omega_y^2 + \frac{1}{2} I_z \omega_z^2$$

Example 12 The natural frequency of the system is designed to be ω_n = 10 rad/s. The spring constant k_2 is half of k_1, and the mass is 1 kg. The mass associated with the other components may be assumed to be negligible. For the given natural frequency, the spring constant k_1 is most nearly

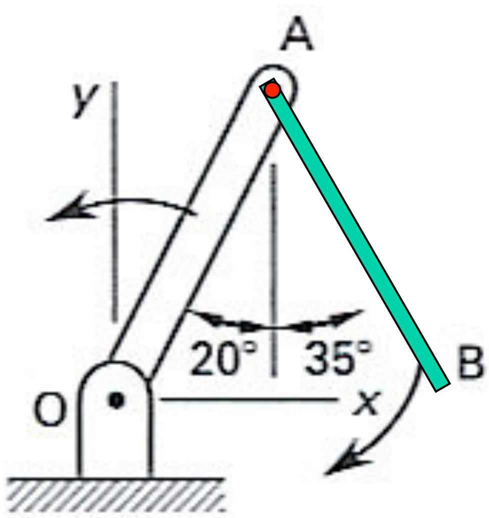
$$m\ddot{x} + k_{eq}x = 0$$

$$\frac{1}{k_{eq}} = \frac{1}{2k_1} + \frac{1}{k_2}$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$



Example 13 A two-bar linkage rotates about the pivot point O as shown. The length of members AB and OA are 2.0 m and 2.5 m, respectively. The angular velocity and acceleration of member OA is ω_{OA} = 0.8 rad/s CCW and α_{OA} = 0 rad/s^2. The angular velocity of member AB is ω_{AB} = 1.2 rad/s CW, and the acceleration of member AB is α_{AB} = 3 rad/s^2 CCW. When the bars are in the position shown, the magnitude of the acceleration of point B is most nearly



$$\vec{a}_A = \vec{\omega}_{OA} \times \left(\vec{\omega}_{OA} \times \vec{r}_{A/O}\right) + \vec{\alpha}_{OA} \times \vec{r}_{AO}$$

$$\vec{a}_B = \vec{a}_A + \vec{\omega}_{AB} \times \left(\vec{\omega}_{AB} \times \vec{r}_{B/A}\right) + \vec{\alpha}_{AB} \times \vec{r}_{B/A} + 2\vec{\omega}_{AB} \times \vec{v}_{B/A} + \vec{a}_{B/A}$$

Gears – Quick Review

Spur Gears

Helical Gears

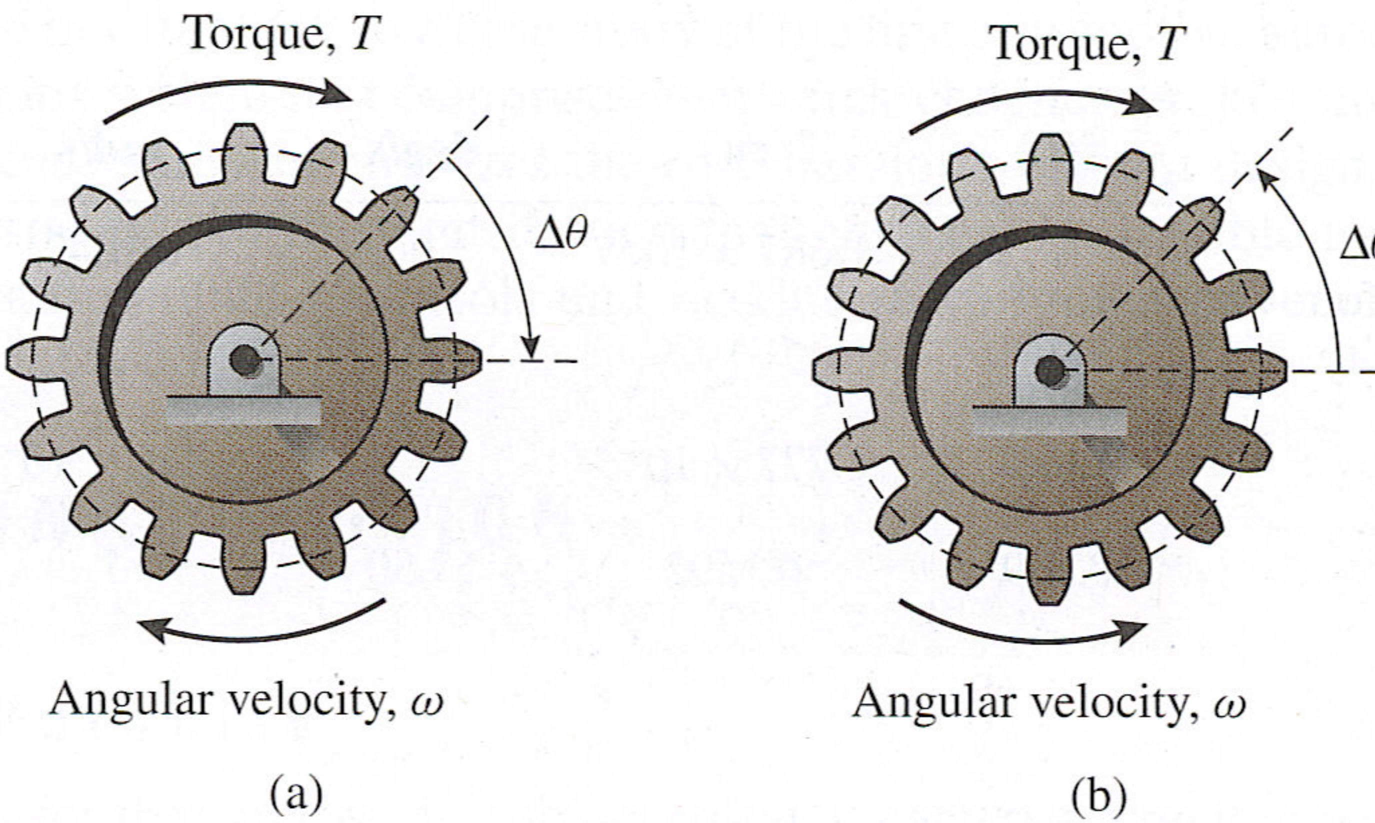
Bevel Gears

Types of Gears

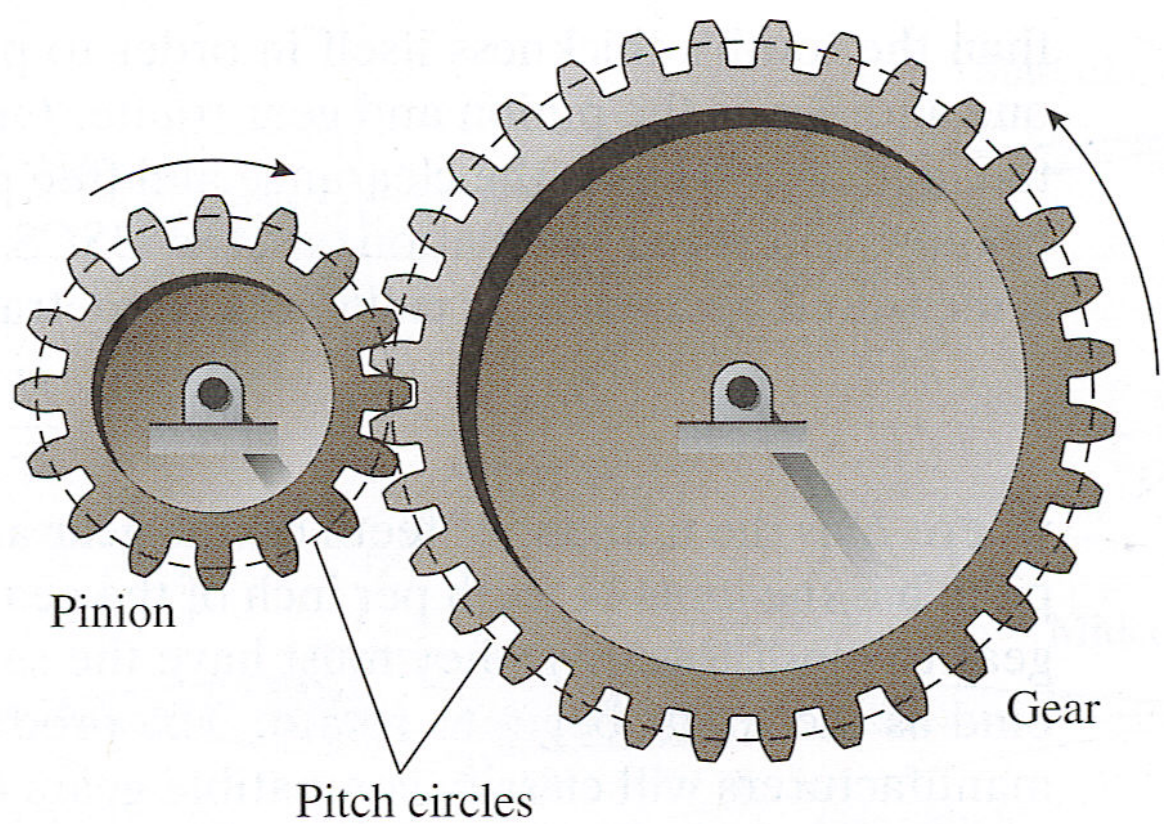
Worm Gear

Rack and Pinion

Concepts Review

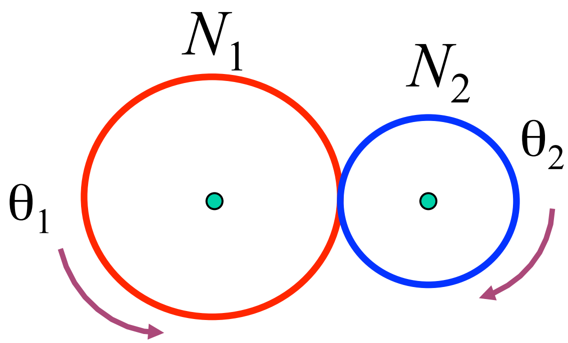


Terminology



Fundamental Law of Gearing

Fixed Shafts



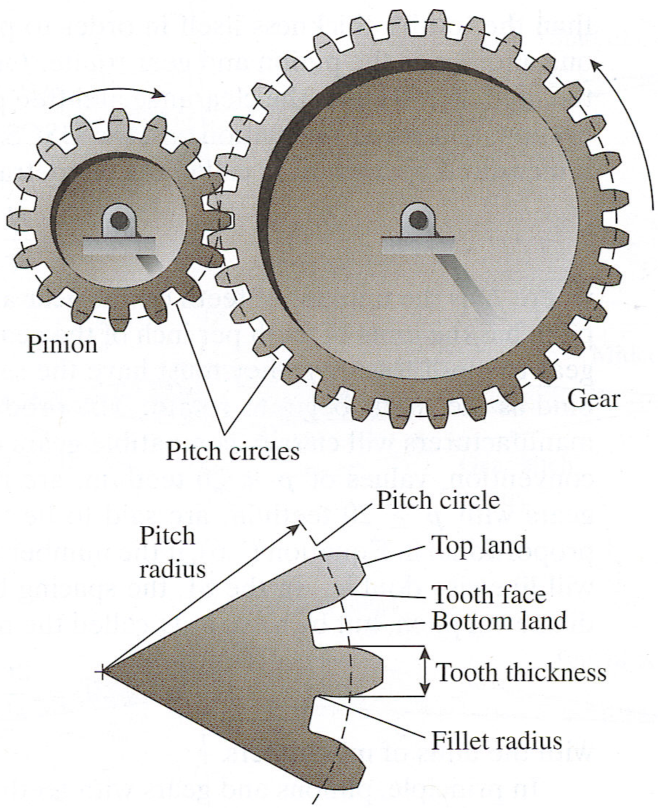
Gear Ratio

$$N = \frac{r_2}{r_1} = \frac{N_2}{N_1}$$

$$r_1\theta_1 = r_2\theta_2$$

$$N = \frac{r_2}{r_1} = \frac{N_2}{N_1} = \frac{\theta_1}{\theta_2} = \frac{\omega_1}{\omega_2}$$

Gear Compatibility



USCS

$$p = \frac{N}{2r}$$

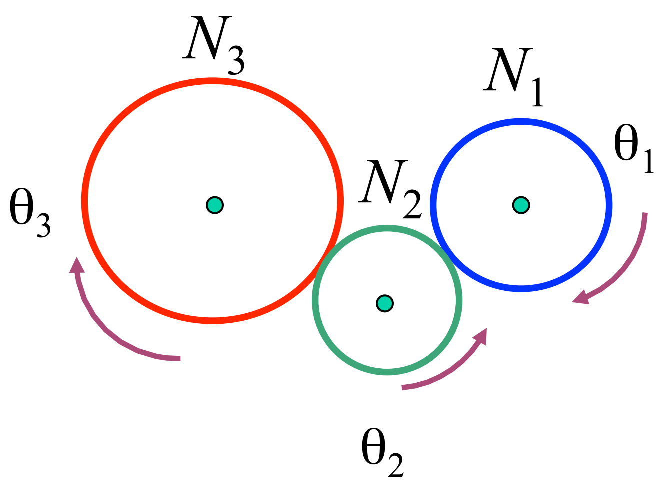
Units???

SI

$$m = \frac{2r}{N}$$

Idlers

Fixed Shafts



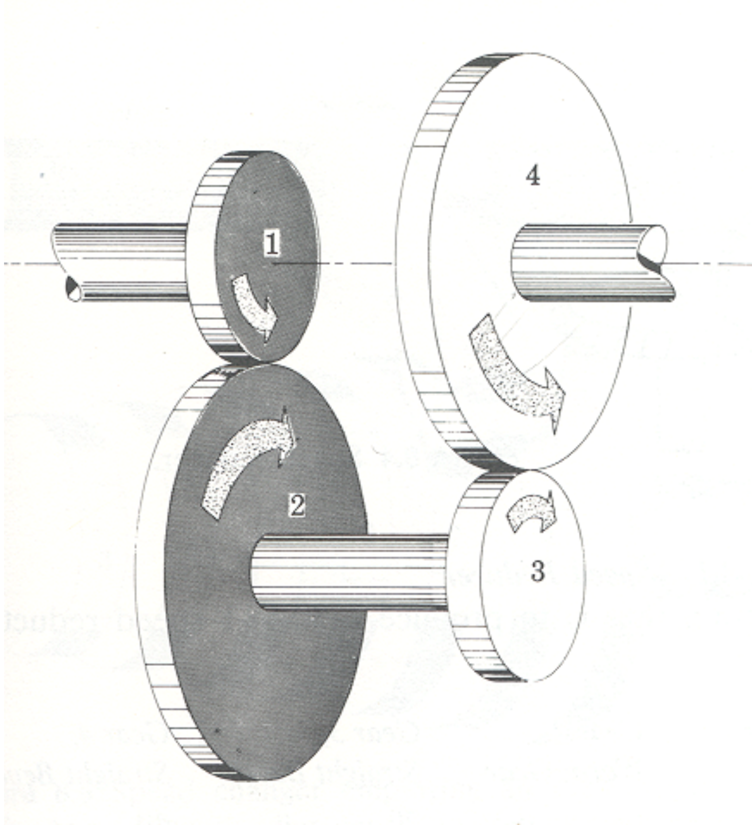
Gear Ratio

$$N = \frac{r_2}{r_1} = \frac{N_2}{N_1} = \frac{\theta_1}{\theta_2} = \frac{\omega_1}{\omega_2}$$

$$\frac{\omega_3}{\omega_1} = \frac{\omega_3}{\omega_2}\frac{\omega_2}{\omega_1} = \frac{N_2}{N_3}\frac{N_1}{N_2} = \frac{N_1}{N_3}$$

Double Reductions

Fixed Shafts



Gear Ratio

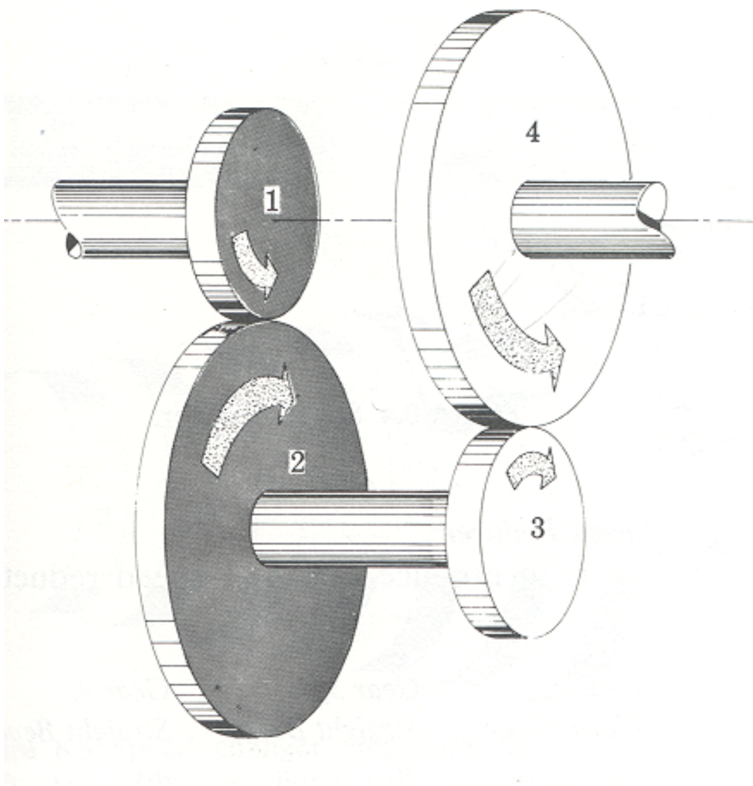
$$n_3 = n_2 = -n_1\left(\frac{N_1}{N_2}\right)$$

$$n_4 = -n_3\left(\frac{N_3}{N_4}\right)$$

$$\frac{n_4}{n_1} = \frac{N_1 N_3}{N_2 N_4}$$

Double Reductions

Fixed Shafts



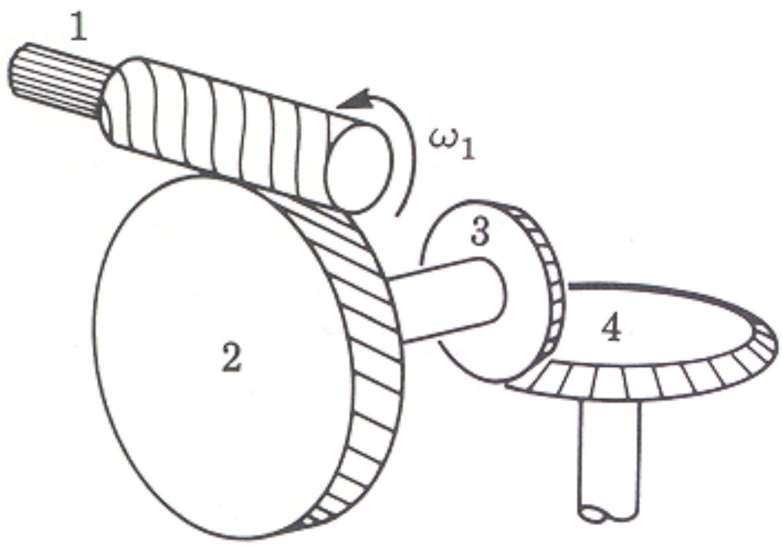
Gear Ratio

$$\frac{n_{\text{output}}}{n_{\text{input}}} = \frac{\text{product of driving teeth}}{\text{product of driven teeth}}$$

$$\frac{n_4}{n_1} = \frac{N_1 N_3}{N_2 N_4}$$

Example 1 - Speed Reducer

	Gear 1, Worm	*Gear 2 Worm Gear*	*Gear 3, Straight Bevel*	*Gear 4, Straight Bevel*
Tooth numbers	2		20	40
rev/min	1000			20

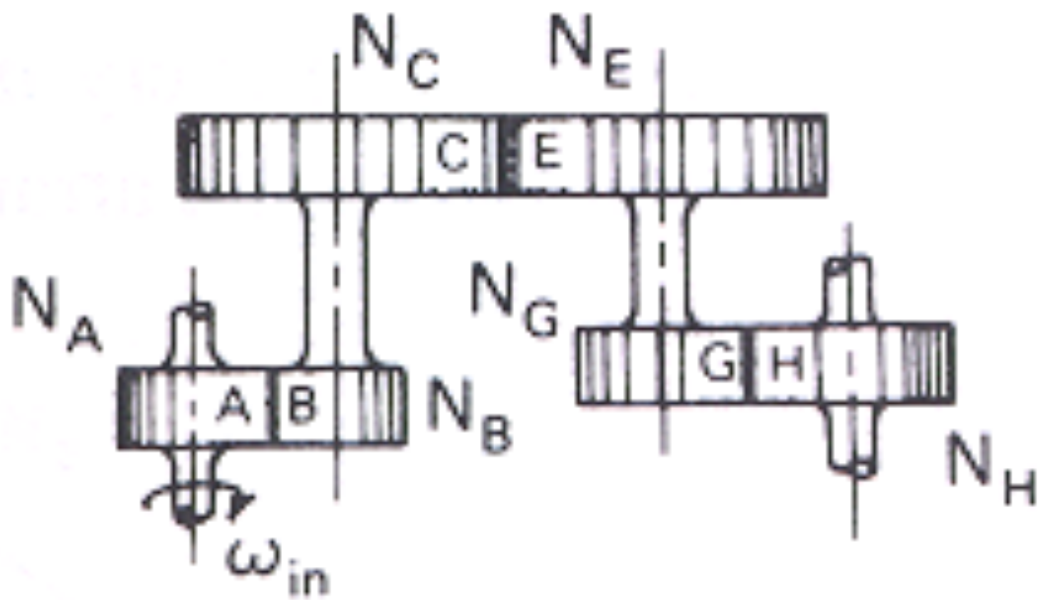


$$N_2 = 50$$

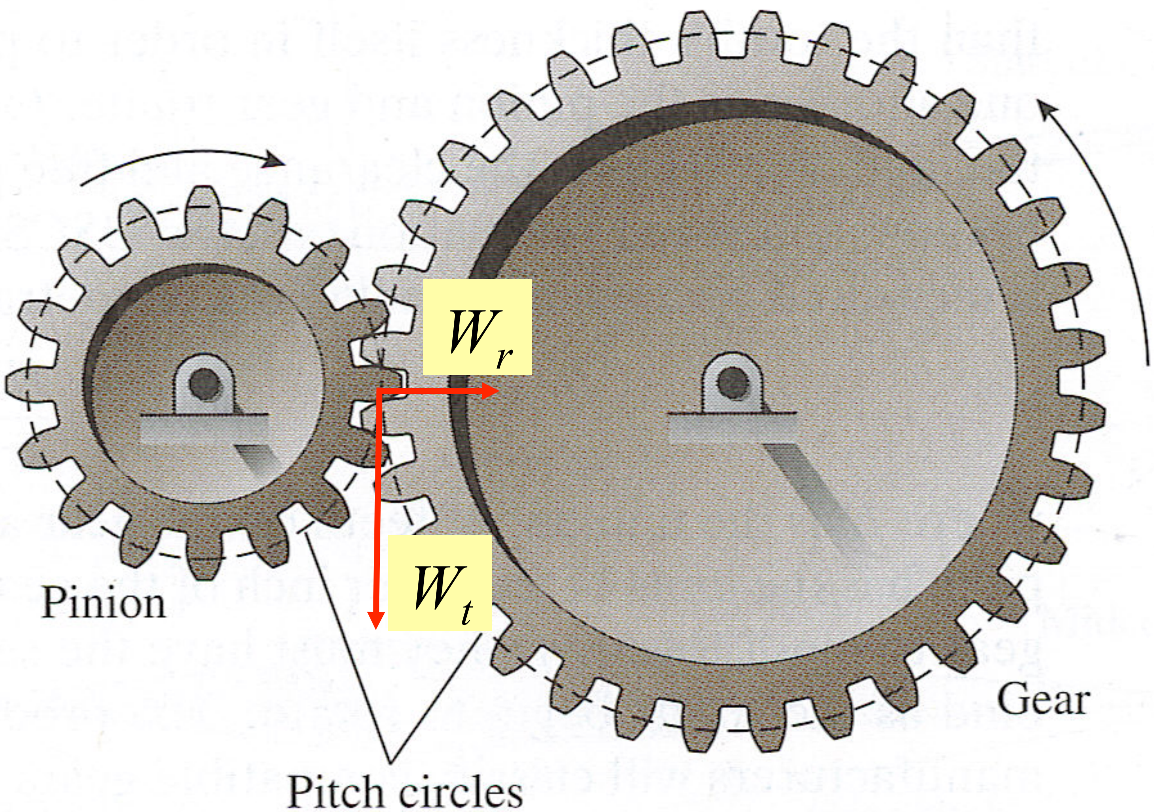
$$n_2 = n_3 = 40 \ \frac{\text{rev}}{\text{min}}$$

Example 2

The compound gear train shown is attached to a motor that drives gear *A* at ω in clockwise as viewed from below. What is the expression for the angular velocity of gear H in terms of the number of teeth on the gears? What is the direction of rotation of gear H as viewed from below.



Gear Forces



$$\tan\phi = \frac{W_r}{W_t}$$

$$W_t = \frac{2T}{d} = \frac{2T}{mN}$$

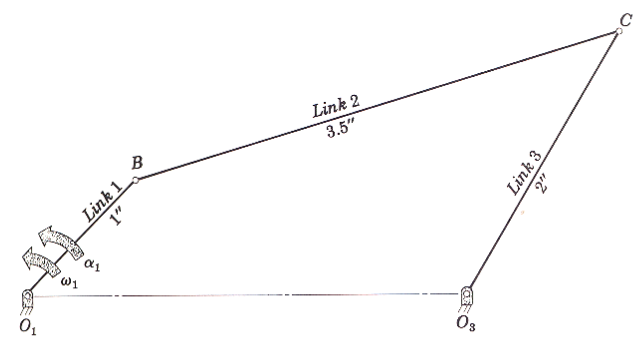
$$W_t = \frac{2H}{d\omega} = \frac{2H}{mN\omega}$$

SI Units: N, mm, kW

Classification of 4-Bar Linkages

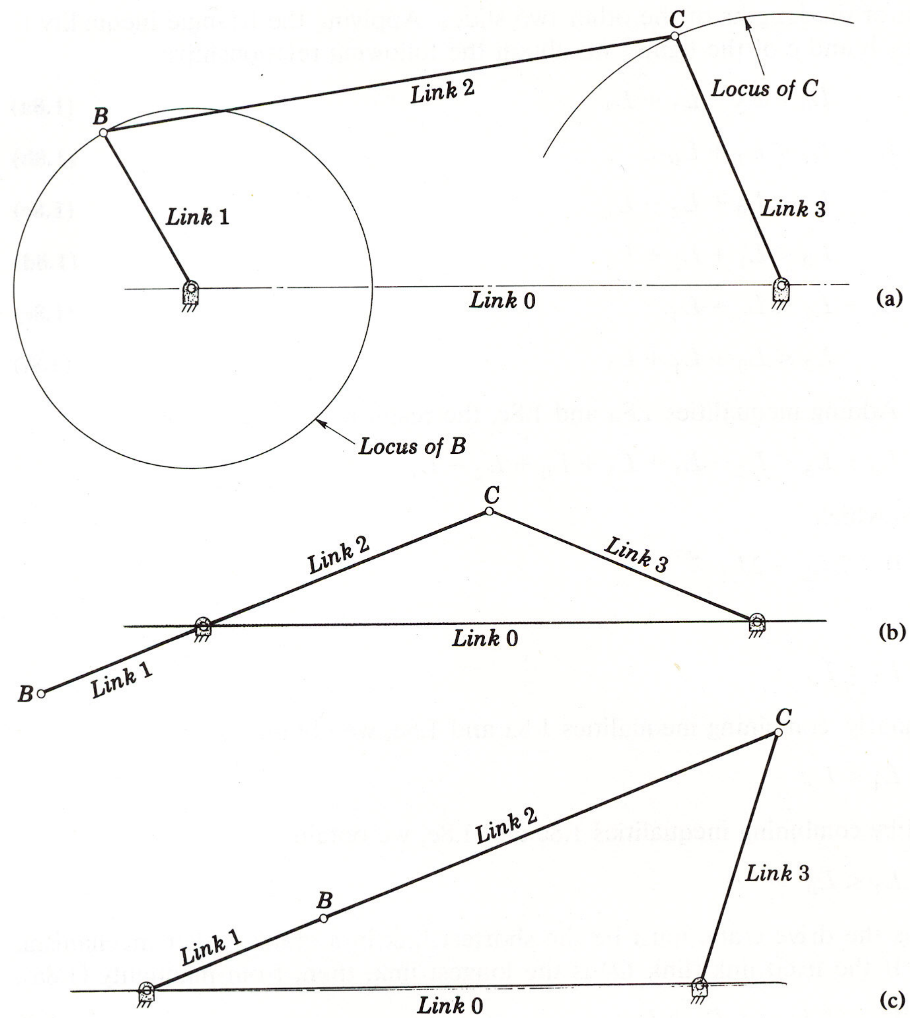
- Grashof Mechanism – One link can perform a full rotation relative to another link
- Grashof Criterion

$$L_{\max} + L_{\min} < L_a + L_b$$



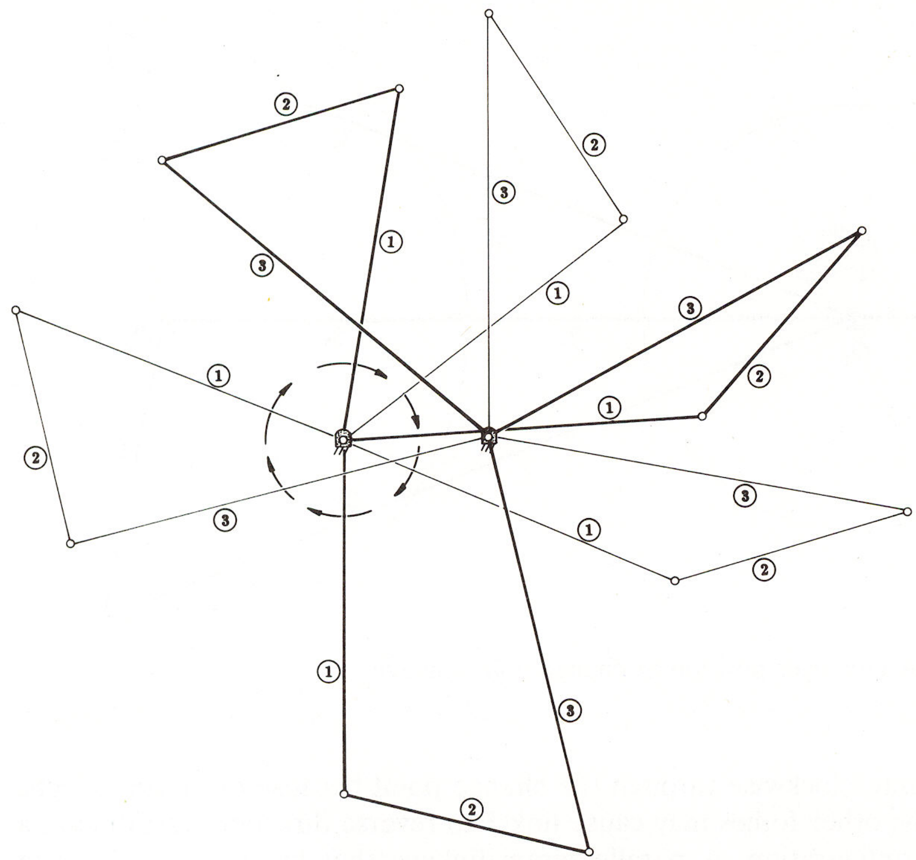
Crank-Rocker Mechanism

$$L_1 = L_{min}$$



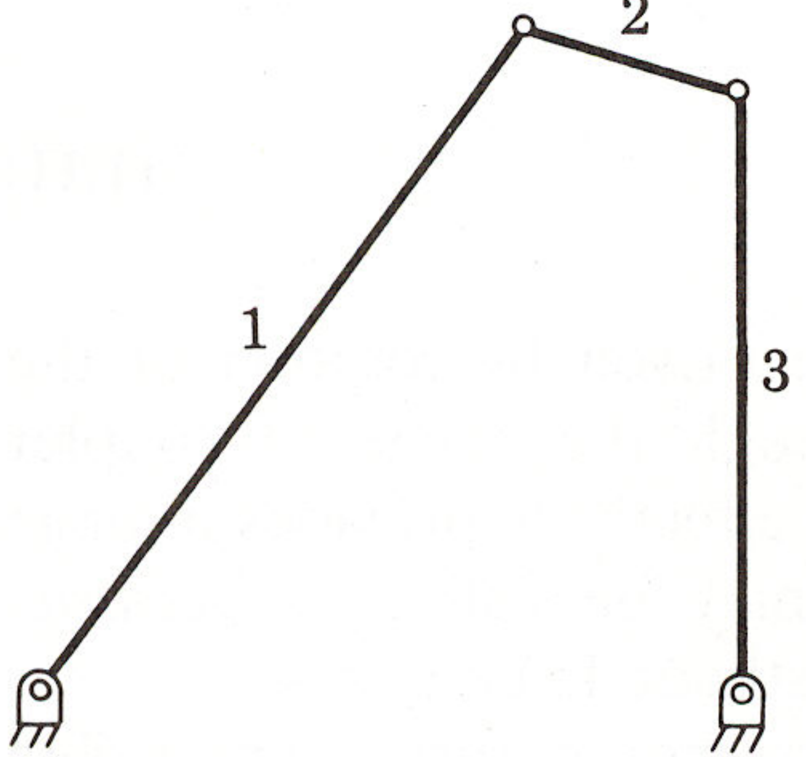
Drag Link Mechanism

$$L_0 = L_{min}$$



Double-Rocker Mechanism

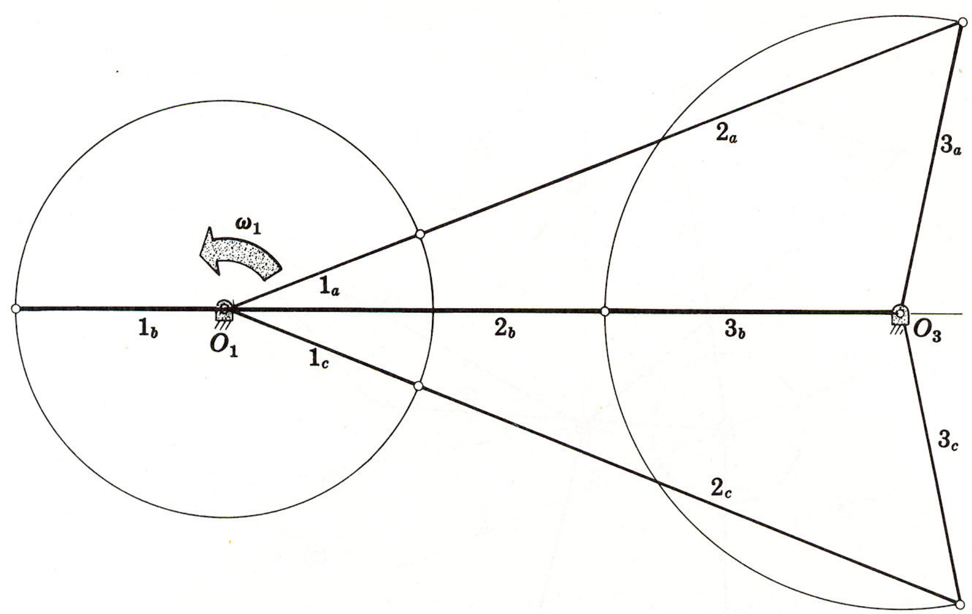
$$L_2 = L_{min}$$



Change-Point Mechanism

Also called Crossover-Position Mechanism

$$L_{\max} + L_{\min} = L_a + L_b$$



Non-Grashof Mechanism

- No link can rotate through 360°
- Double Rocker Mechanism of the 2nd Kind
- Triple-Rocker Mechanism

$$L_{\max} + L_{\min} > L_a + L_b$$

TABLE 1.1 SUMMARY OF THE CRITERIA OF MOTION FOR EACH CLASS OF FOUR-BAR LINKAGES

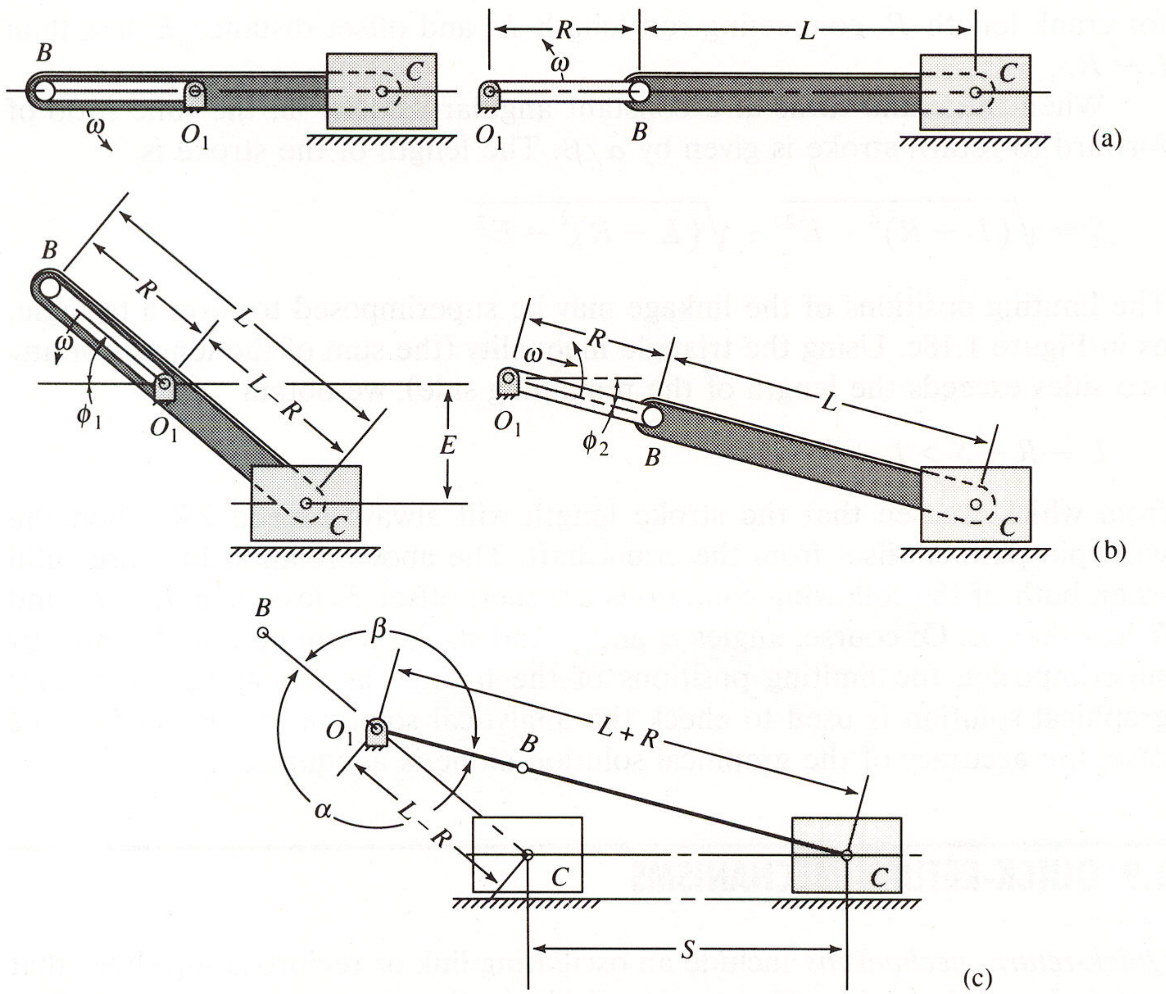
L_{min}: shortest link
L_{max}: longest link
L_a and L_b: links of intermediate length

Type of mechanism	Shortest link	Relationship between link lengths
Grashof	Any	$L_{max} + L_{min} \leq L_a + L_b$
Crank rocker	Driver crank*	$L_{max} + L_{min} < L_a + L_b$
Drag link	Fixed link	$L_{max} + L_{min} < L_a + L_b$
Double rocker	Coupler	$L_{max} + L_{min} < L_a + L_b$
Crossover-position change point	Any	$L_{max} + L_{min} = L_a + L_b$
Non-Grashof		
Double rocker of the second kind (triple rocker)	Any	$L_{max} + L_{min} > L_a + L_b$†

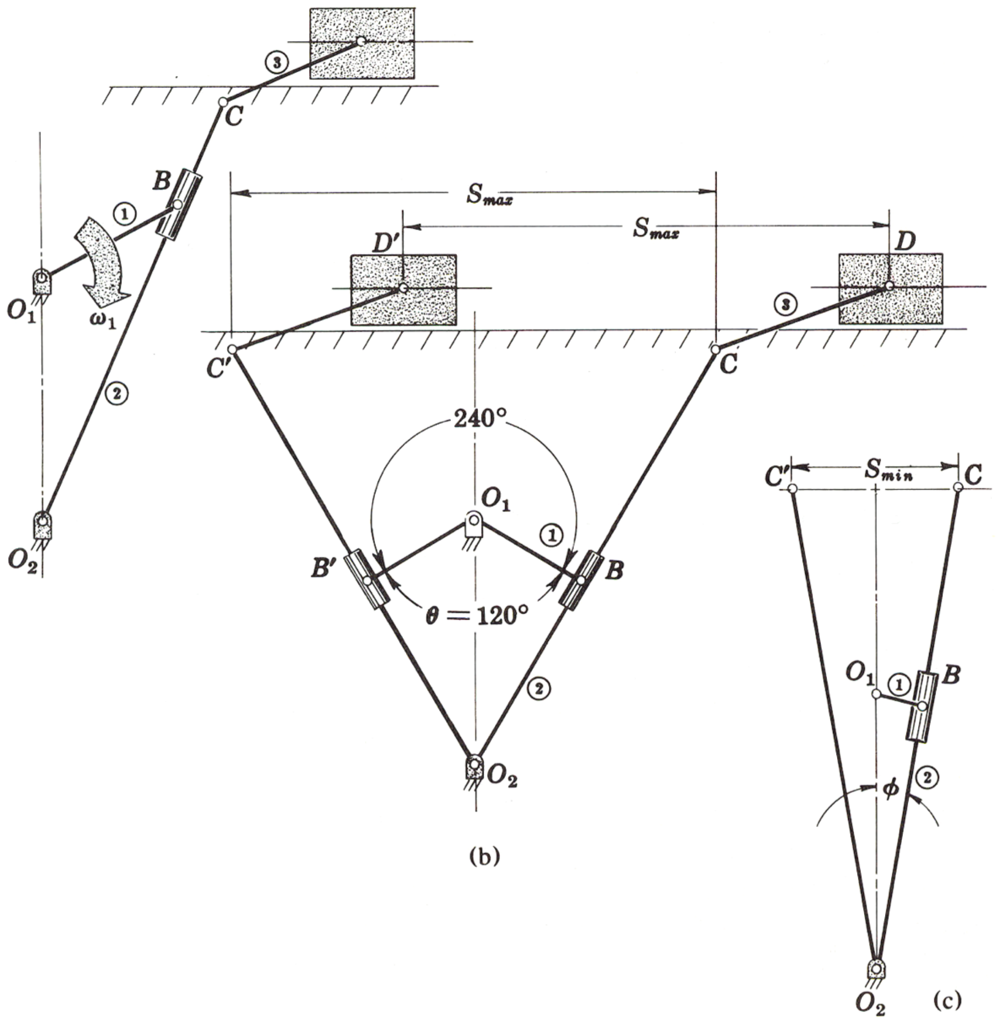
*If the driven crank is the shortest link, its direction of motion is uncertain when the driver crank is at a limiting position.

†$L_{max} < L_{min} + L_a + L_b$ (otherwise, a four-bar linkage cannot be constructed).

Crank-Slider Mechanism



Quick Return Mechanism



Geneva Mechanism