CE303 Introduction to Construction Engineering

Engineering Economics

Engineering Economics

- Study of the desirability of making an investment
- Very little, if any, true economics (micro or macro) in this subject.

The Application Of Engineering Economics

- Cash Flow
- Time Value Of Money
- Equivalence
- Compound Interest
- Single Payment Formulas
- Uniform Payment Series Formulas

The Application Of Engineering Economics

- Uniform Gradient
- Continuous Compounding
- Nominal And Effective Interest
- Present Worth Analysis
- Indefinite Life And Capitalized Cost

The Application Of Engineering Economics

- Future Worth Or Value
- Annual Cost
- Rate Of Return Analysis
- Benefit-cost Analysis

The Application Of Engineering Economics

- Valuation And Depreciation
- Straight Line Depreciation
- Modified Accelerated Cost Recovery System Depreciation Inflation
- Effect Of Inflation On A Rate Of Return

Interest & Interest Rate

- **Interest**
  - A fee assessed to use borrowed money.
  - The size of the fee will depend upon the amount of money borrowed and the length of time which it is borrowed.
- **Interest Rate**
  - The percentage rate charged as interest.

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Simple Interest

A fixed percentage of the principal multiplied by the life of the loan.

If:

I = total amount of simple interest

n = life of the loan

i= interest rate (expressed as a decimal)

P = principal

Then:

$I = niP$

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Interest Example Problem

$100,000 was deposited in a bank account and $115,000 is withdrawn one year later.

Compute

a) the interest received from the $100,000 investment, and
b) the annual interest rate which was paid.

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Interest Example Problem

A)

$$I = \$115,000 - \$100,000 = \$15,000$$

B)

$$i = \frac{\$15,000/\text{yr}}{\$100,000} \times 100\% = 15\% \text{ per year}$$

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Example 1

- At a 10% per year interest rate, how much is $500 now equivalent to three years from now?
- $500 now will increase 10% in each on the three years.

Now	End of 1st year	End of 2nd year	End of 3rd year
$500.00	500 + 10%(500)	550 + 10%(550)	605 + 10%(605)
	$550.00	$605.00	$665.50

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Compound Interest

Interest is computed and credited at the end of each interest period, and is allowed to accumulate from one interest period to the next.

$$F = P(1+i)^n$$

F = the total amount of money accumulated

P = Present Value of Money
i = interest rate (decimal form)
n = Number of interest periods

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The Time Value of Money

- Which would you prefer,
  A. $100 today or
  B. $105 a year from now?

- If you had the $100 today, you could use it for the year. If you had no use for it now, you could lend and receive interest for the privilege of using your money for the year.

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The Time Value of Money

- Money has the ability to earn interest.
- Its value increases with time.
- Since money increases as we move forward from the present to the future, it also must decrease in value if we move backward from the future to the present.

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Cash Flows

- The difference between total cash receipts (***inflows***) and total cash disbursements (***outflows***) for a given period of time.
- Important concept in engineering economics because they form the basis for evaluating projects, equipment and investing alternatives.

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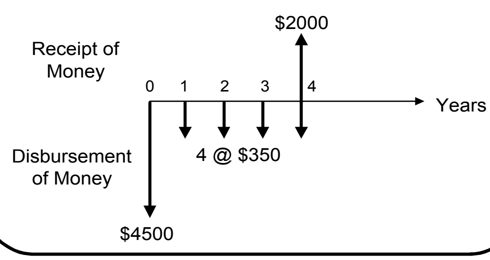
Cash Flows

- A cash flow table shows "***Cash Flow***" and "***Timing***."

Year	Cash Flow	Comment
Beginning of Year 1	-$4500	Car is purchased "now" for $4500
End of Year 1	-$350	Maintenance cost per year
End of Year 2	-$350	Maintenance cost per year
End of Year 3	-$350	Maintenance cost per year
End of Year 4	-$350 +$2000	Maintenance cost per year The car is sold for $2000

Cash Flows ~ Graphically



Example 2

- In January 2004 a firm purchased a used plotter for $500.
- In 2005 there were no repairs necessary.
- In 2006, 2007, and 2008, repair costs were $85, $130, and $140, respectively.
- The plotter is sold in 2008 for $300.
- Develop the cash flow table.

Example 2

Normal Conventions

- Purchases are at the beginning-of-year,
- Disbursements & receipts are at the end-of-year,
- Resale or salvage value is at the end-of-year.
- Repairs & resale are at the end-of-year.
- A negative sign represents a disbursement and a positive sign represents a receipt.

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Example 2

	Year	Cash Flow
Beginning of 2003	0	-$500
End of 2003	1	0
End of 2004	2	0
End of 2005	3	-$85
End of 2006	4	-$130
End of 2007	5	-$140 + $300 = +$160

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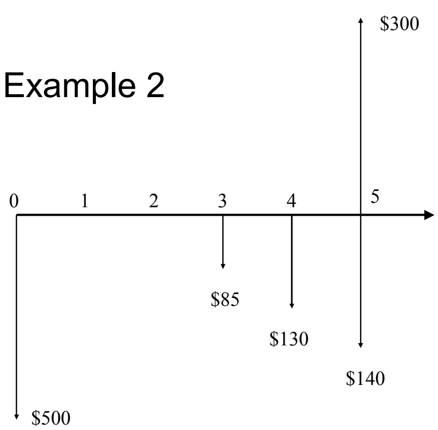
Example 2



Equivalence

- Money at different points in time may have the same value in that they may both be worth the same amount in today's dollars.
- When alternatives are acceptable substitutes, they are said to be equivalent.
- For example, at an 8% interest rate, $100 today is equivalent to $108 a year from now.

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Example 1 Revisited

- At a 10% per year interest rate, how much is $500 now equivalent to three years from now?
- $500 now will increase 10% in each on the three years.

Now	End of 1st year	End of 2nd year	End of 3rd year
$500.00	500 + 10%(500)	550 + 10%(550)	605 + 10%(605)
	$550.00	$605.00	$665.50

- The $500 now is equivalent to $665.50 at the end of three years.

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Equivalence in Engineering Economics

If we wish to select the better of two alternatives,

First, we have to compute the cash flows.

Year	Alternative A	Alternative B
0	-$2000	-$2800
1	+$800	+$1100
2	+$800	+$1100
3	+$800	+$1100

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Compound Interest Formulas

SINGLE PAYMENT

Factor	To Find	Given	Function Name	Formula
Compound Amount	F	P	(F/P, i, n)	$F = P(1+i)^n$
Present Worth	P	F	(P/F, i, n)	$P = F(1+i)^{-n}$

UNIFORM PAYMENT SERIES

Factor	To Find	Given	Function Name	Formula
Sinking Fund	A	F	(A/F, i, n)	$A = F\left[\frac{i}{(1+i)^n - 1}\right]$
Capital Recovery	A	P	(A/P, i, n)	$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$
Compound Amount	F	A	(F/A, i, n)	$F = A\left[\frac{(1+i)^n - 1}{i}\right]$
Present Worth	P	A	(P/A, i, n)	$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

Compound Interest Notation

- ***i*** ≡ Effective interest rate per interest period.
  In equations, the interest rate is stated as a decimal (that is, 8% interest is 0.08)
- ***n*** ≡ Number of interest periods.
  The interest period is usually one year, but may be different.
- ***P*** ≡ the present sum of money.
- ***F*** ≡ the future sum of money or an amount at an interest rate ***i, n*** interest periods from present that is equivalent to ***P***.

Compound Interest Notation

- ***A*** ≡ An end-of-period receipt/disbursement of a uniform series continuing for ***n*** periods. The entire series is equivalent to a ***P*** or a ***F*** at interest rate ***i***.
- ***G*** ≡ Uniform period-by-period increase in cash flows, the uniform gradient.
- The functional notation scheme is based on the expression ***(Find/Given, i, n)***

Example 1

- At a 10% per year interest rate, how much is $500 now equivalent to three years from now?
- $500 now will increase 10% in each on the three years.

Now	End of 1st year	End of 2nd year	End of 3rd year
$500.00	500 + 10%(500)	550 + 10%(550)	605 + 10%(605)
	$550.00	$605.00	$665.50

- The $500 now is equivalent to $665.50 at the end of three years.

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Single Payment Formulas

- Suppose a present sum of money ***P*** is invested for one year at an interest rate of ***i***%.
- At the end of one year, the initial investment ***P*** is received together with interest equal to ***Pi*** or a total amount ***P*** + ***Pi***, or ***P(1 + i).***

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Single Payment Formulas

- If the investment is allowed to remain for subsequent years, the progression is:

Amount at Beginning Of Period	+	*Interest for The period*	=	*Amount at End Of the Period*
1^{st} *year* P	+	Pi	=	$P(1+i)$
2^{nd} *year* $P(1+i)$	+	$Pi(1+i)$	=	$P(1+i)^2$
3^{rd} *year* $P(1+i)^2$	+	$Pi(1+i)^2$	=	$P(1+i)^3$
n^{th} *year* $P(1+i)^{n-1}$	+	$Pi(1+i)^{n-1}$	=	$P(1+i)^n$

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Single Payment Compound Amount Formula

- The present sum ***P*** increases in ***n*** periods to $P(1+i)^n$.
- This gives the relation between a present sum ***P*** and its equivalent future sum ***F***.
- F = Present Sum $x\ (1+i)^n = P(1+i)^n$
- In functional notation it is written:

$$F = P(F/P, i, n)$$

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Single Payment Present Worth Factor

- The reciprocal of the single payment, compound amount factor

$$P/F = (F/P)^{-1} = (1+i)^{-n}$$

- The notation (***P/F, i%,n***) (translation: Given a sum of money, F, earns interest at a rate ***i***, compounded annually" (if n is in years)).

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Single Payment Present Worth Formula

- This relationship may be rewritten as:

Present Sum = P = Future Sum $x\ (1+i)^{-n}$

$$= F(1+i)^{-n}$$

- In functional notation, it is written:

$$P = F(P/F, i, n)$$

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Example 3

- At a 10% per year interest rate, $500 now is equivalent to how much three years from now?
- Solution: It can be solved using a single payment, compound amount factor.
- Given:

  P = $500, ***n*** = 3 years, ***i*** = 10%, & ***F*** = unknown

  Find F:

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Example 4

To raise capital for his new business, a man asks you to lend him money. He offers to pay you $3,000 at the end of four years. How much should you give him now if you want to make 12% interest per year?

Solution:

P = unknown, ***F*** = $3000, ***n*** = 4 years, & ***i*** = 12%

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Example 4

- Using a compound interest table:
- ***P = F(P/F, i, n)*** = 3000(***P/F***,12%, 4) =
  = 3000(0.6355) =
  $1,906.50
- The solution based on the compound interest table is slightly different from the solution using a calculator.
- The compound interest tables are considered to be sufficiently accurate to solve engineering economic problems.

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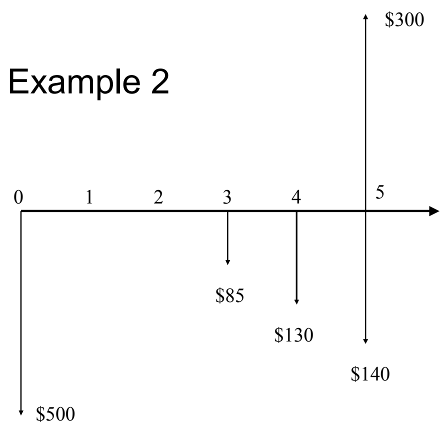
Factor Table - *i* = 12.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.8929	0.8929	0.0000	1.1200	1.0000	1.1200	1.0000	0.0000
2	0.7972	1.6901	0.7972	1.2544	2.1200	0.5917	0.4717	0.4717
3	0.7118	2.4018	2.2208	1.4049	3.3744	0.4163	0.2963	0.9246
4	0.6355	3.0373	4.1273	1.5735	4.7793	0.3292	0.2092	1.3589
5	**0.5674**	**3.6048**	**6.3970**	**1.7623**	**6.3528**	**0.2774**	**0.1574**	**1.7746**
6	0.5066	4.1114	8.9302	1.9738	8.1152	0.2432	0.1232	2.1720
7	0.4523	4.5638	11.6443	2.2107	10.0890	0.2191	0.0991	2.5515
8	0.4039	4.9676	14.4714	2.4760	12.2997	0.2013	0.0813	2.9131
9	0.3606	5.3282	17.3563	2.7731	14.7757	0.1877	0.0677	3.2574
10	**0.3220**	**5.6502**	**20.2541**	**3.1058**	**17.5487**	**0.1770**	**0.0570**	**3.5847**
11	0.2875	5.9377	23.1288	3.4785	20.6546	0.1684	0.0484	3.8953
12	0.2567	6.1944	25.9523	3.8960	24.1331	0.1614	0.0414	4.1897
13	0.2292	6.4235	28.7024	4.3635	28.0291	0.1557	0.0357	4.4683
14	0.2046	6.6282	31.3624	4.8871	32.3926	0.1509	0.0309	4.7317
15	**0.1827**	**6.8109**	**33.9202**	**5.4736**	**37.2797**	**0.1468**	**0.0268**	**4.9803**
16	0.1631	6.9740	36.3670	6.1304	42.7533	0.1434	0.0234	5.2147
17	0.1456	7.1196	38.6973	6.8660	48.8837	0.1405	0.0205	5.4353
18	0.1300	7.2497	40.9080	7.6900	55.7497	0.1379	0.0179	5.6427
19	0.1161	7.3658	42.9979	8.6128	63.4397	0.1358	0.0158	5.8375
20	**0.1037**	**7.4694**	**44.9676**	**9.6463**	**72.0524**	**0.1339**	**0.0139**	**6.0202**
21	0.0926	7.5620	46.8188	10.8038	81.6987	0.1322	0.0122	6.1913
22	0.0826	7.6446	48.5543	12.1003	92.5026	0.1308	0.0108	6.3514
23	0.0738	7.7184	50.1776	13.5523	104.6029	0.1296	0.0096	6.5010
24	0.0659	7.7843	51.6929	15.1786	118.1552	0.1285	0.0085	6.6406
25	**0.0588**	**7.8431**	**53.1046**	**17.0001**	**133.3339**	**0.1275**	**0.0075**	**6.7708**
30	0.0334	8.0552	58.7821	29.9599	241.3327	0.1241	0.0041	7.2974
40	0.0107	8.2438	65.1159	93.0510	767.0914	0.1213	0.0013	7.8988
50	0.0035	8.3045	67.7624	289.0022	2,400.0182	0.1204	0.0004	8.1597
60	0.0011	8.3240	68.8100	897.5969	7,471.6411	0.1201	0.0001	8.2664
100		**8.3332**	**69.4336**	**83,522.2657**	**696,010.5477**	**0.1200**		**8.3321**

Example 2



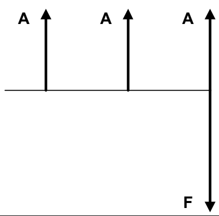
Single Payment Compound Amount Formula

- The present sum ***P*** increases in ***n*** periods to $P(1+i)^n$.
- This gives the relation between a present sum ***P*** and its equivalent future sum ***F***.
- F = Present Sum x $(1+i)^n = P(1+i)^n$
- In functional notation it is written:

  $F = P(F/P, i, n)$

Uniform Payment Series Formulas

Consider



A = End-of-period cash receipt or disbursement in a uniform series continuing for n periods

F = A future sum of money

Uniform Series Formulas

- Using the single payment compound amount factor, we can write an equation from F in terms of A:
- $F = A + A(1+i) + A(1+i)^2$
- In this situation, with n = 3, the equation may be written in a more general form:
- $F = A + A(1+i) + A(1+i)^{n-1}$

Uniform Series Compound Amount Formula

- Multiply the first equation by $(1+i)$,
  $(1+i)F = A(1+i) + A(1+i)^{n-1} + A(1+i)^n$
- Subtracting the second equation:
  $(1+i)F = A(1+i) + A(1+i)^{n-1} + A(1+i)^n$
  $-\{F = A + A(1+i) + A(1+i)^{n-1}\}$
  $iF = -A + A(1+i)^n$

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Uniform Series
Compound Amount Factor

- Equal amounts of money, A, is deposited in a savings account at the end of each year.
- The money earns interest at a rate of i, compounded annually.
- To calculate the accumulated money after n years the ratio

$$F/A = \frac{(1+i)^n - 1}{i}$$

- The notation ($F/A, i\%, n$) is helpful setting up the problem, and can be obtained from compound interest tables.

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Uniform Series
Sinking Fund Formula

- Solving the equation for A produces

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

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Uniform Series
Sinking Fund Factor

- The reciprocal of the uniform series, sinking fund factor

$$A/F = (F/A)^{-1} = \frac{i}{(1+i)^n - 1}$$

- The notation ($A/F, i\%, n$) is helpful setting up the problem, and can be obtained from compound interest tables.

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Example 5

- If $100 is deposited at the end of each year in a savings account that pays 6% interest per year,
- How much will be in the account at the end of five years?
- Solution:

  Given:
  A = $100, F = Unknown, n = 5 years, and i = 6%.

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

- F = A(F/A,6%,5) = (100)(?)

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Example 5

- If $100 is deposited at the end of each year in a savings account that pays 6% interest per year,
- How much will be in the account at the end of five years?
- Solution:

  Given:
  A = $100, F = Unknown, n = 5 years, and i = 6%.

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Example 6

- A fund established to produce a desired amount at the end of a given period, by means of a series of payments throughout the period, is called a sinking fund.
- A sinking fund is to be established to accumulate money to replace a $10,000 machine.

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Example 6

- If the machine is to be replaced at the end of 12 years,
- How much should be deposited in the sinking fund each year?
- Assume the fund earns 10% annual interest.
- Solution
  Annual sinking fund deposit
  A = 10,000(A/F, 10%, 12)
  = 10,000(?)

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Example 6

- If the machine is to be replaced at the end of 12 years,
- How much should be deposited in the sinking fund each year?
- Assume the fund earns 10% annual interest.
- Solution
  Annual sinking fund deposit
  A = 10,000(A/F, 10%, 12)
  = 10,000(0.0468) = $468

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Uniform Series Capital Recovery Formula

- Since $F = P(1+i)^n$, we can substitute this expression for F in the equation and obtain the following equation.

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

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Uniform Series Present Worth Formula

- Solving the equation for $\boldsymbol{P}$ produces

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

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Example 7

- An individual is considering the purchase of a used automobile.
- The total price is $6,200.
- With $1,240 as a down payment and the balance paid in 48 equal monthly payments with interest at 1% per month,
- Compute the monthly payment.
- The payments are due at the end of each month.

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Example 7

- Solution:
  The amount to be repaid by the 48 monthly payments is cost of the automobile minus the $1,240 down payment.
- P = $4,960, A = unknown, n = 48 monthly payments, and i = 1% per month.
- A = P(A/P,1%,48) = 4,960(0.0265) = $131.44

4

Factor Table - *i* = 1.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9901	0.9901	0.0000	1.0100	1.0000	1.0100	1.0000	0.0000
2	0.9803	1.9704	0.9803	1.0201	2.0100	0.5075	0.4975	0.4975
3	0.9706	2.9410	2.9215	1.0303	3.0301	0.3400	0.3300	0.9934
4	0.9610	3.9020	5.8044	1.0406	4.0604	0.2563	0.2463	1.4876
5	**0.9515**	**4.8534**	**9.6103**	**1.0510**	**5.1010**	**0.2060**	**0.1960**	**1.9801**
6	0.9420	5.7955	14.3205	1.0615	6.1520	0.1725	0.1625	2.4710
7	0.9327	6.7282	19.9168	1.0721	7.2135	0.1486	0.1386	2.9602
8	0.9235	7.6517	26.3812	1.0829	8.2857	0.1307	0.1207	3.4478
9	0.9143	8.5650	33.6959	1.0937	9.3685	0.1167	0.1067	3.9337
10	**0.9053**	**9.4713**	**41.8435**	**1.1046**	**10.4622**	**0.1056**	**0.0956**	**4.4179**
11	0.8963	10.3676	50.8067	1.1157	11.5668	0.0965	0.0865	4.9005
12	0.8874	11.2551	60.5687	1.1268	12.6825	0.0888	0.0788	5.3815
13	0.8787	12.1337	71.1126	1.1381	13.8093	0.0824	0.0724	5.8607
14	0.8700	13.0037	82.4221	1.1495	14.9474	0.0769	0.0669	6.3384
15	**0.8613**	**13.8651**	**94.4810**	**1.1610**	**16.0969**	**0.0721**	**0.0621**	**6.8143**
16	0.8528	14.7179	107.2734	1.1726	17.2579	0.0679	0.0579	7.2886
17	0.8444	15.5623	120.7834	1.1843	18.4304	0.0643	0.0543	7.7613
18	0.8360	16.3983	134.9957	1.1961	19.6147	0.0610	0.0510	8.2323
19	0.8277	17.2260	149.8950	1.2081	20.8109	0.0581	0.0481	8.7017
20	**0.8195**	**18.0456**	**165.4664**	**1.2202**	**22.0190**	**0.0554**	**0.0454**	**9.1694**
21	0.8114	18.8570	181.6950	1.2324	23.2392	0.0530	0.0430	9.6354
22	0.8034	19.6604	198.5663	1.2447	24.4716	0.0509	0.0409	10.0998
23	0.7954	20.4558	216.0660	1.2572	25.7163	0.0489	0.0389	10.5626
24	0.7876	21.2434	234.1800	1.2697	26.9735	0.0471	0.0371	11.0237
25	**0.7798**	**22.0232**	**252.8945**	**1.2824**	**28.2432**	**0.0454**	**0.0354**	**11.4831**
30	0.7419	25.8077	355.0021	1.3478	34.7849	0.0387	0.0277	13.7557
40	0.6717	32.8347	596.8561	1.4889	48.8864	0.0305	0.0205	18.1776
50	0.6080	39.1961	879.4176	1.6446	64.4632	0.0255	0.0155	22.4363
60	0.5504	44.9550	1,192.8061	1.8167	81.6697	0.0222	0.0122	26.5333
100	**0.3697**	**63.0289**	**2,605.7758**	**2.7048**	**170.4814**	**0.0159**	**0.0059**	**41.3426**

Uniform Series Factor Summary

Compound Amount	***(F/A, i, n)***
Sinking Fund	***(A/F, i, n)***
Capital Recovery	***(A/P, i, n)***
Present Worth	***(P/A, i, n)***

Nominal And Effective Interest

- Nominal interest is the annual interest rate without considering the effect of any compounding.
- Effective interest is the annual interest rate taking into account the effect of any compounding during the year.

Nominal And Effective Interest

- Unless specifically qualified in the problem, the interest rate given is an annual rate
- If the compounding is annual, the rate given is the effective rate. If compounding is other than annual, the rate given is the nominal rate.

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Effective Interest Rates

- Effective Annual Interest Rate

$$i_e = (1+r/m)^m - 1$$

- r = Nominal Annual Interest Rate
- m = Number of Compound Periods per year
- r/m = Effective Interest Rate per period

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Example 9

- A bank charges 1½ % interest rate per month on the unpaid balance for purchases made on its credit card.
- What is the nominal interest rate that the bank is charging?
- What is the effective annual interest rate?

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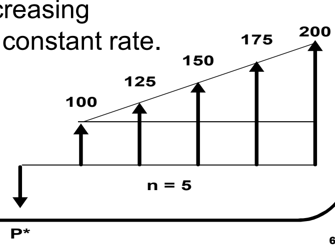
Example 9

- Solution:
  The nominal interest rate is simply the annual interest ignoring compounding.
  - i = 12 x 1½% = 18%
- The effective annual interest rate is computed considering compounding.
  - i_e= (1+(0.18/12))^{12}-1 = 0.1956 = 19.56%

Uniform Gradient Series

- Cash flow series may not always be a constant amount A.
- It may be increasing
  at a constant rate.



Gradient Series Factors

A gradient series is a series of annual payments in which each payment is greater than the previous one by a constant amount, G.

$$A = G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

$$A/G = \frac{1}{i} - \frac{n}{(1+i)^n - 1}$$

$$(A/G, i\%, n) = \frac{1}{i} - \frac{n}{i}(A/F, i\%, n)$$

Gradient Series
Compound Interest

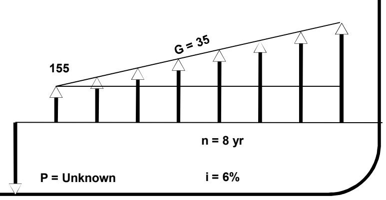
UNIFORM GRADIENT

Factor	To Find	Given	Function Name	Formula
Gradient Present Worth	P	G	(P/G, i, n)	$P = G\left[\frac{(1+i)^n - 1}{i^2(1+i)^n} - \frac{n}{i(1+i)^n}\right]$
Gradient Future Worth	F	G	(F/G, i, n)	$F = G\left[\frac{(1+i)^n - 1}{i^2} - \frac{n}{i}\right]$
Gradient Uniform Series	A	G	(A/G, i, n)	$A = G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$

Example 10

- The maintenance on a machine is expected to be $155 at the end of the first year, and it is expected to increase $35 each year for the following seven years.
- What sum of money should be set aside to pay the maintenance for the eight-year period?
- Using a 6% interest.

Example 10



Example 10

- Note the diagram for the uniform gradient factors.
- The first term in the uniform gradient is zero and the last term is ***(n-1)G***.
- But n is used in the equations and function notation.
- The derivations were done on this basis, and the uniform gradient compound interest tables are computed this way.

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Example 11

- For the situation in Example 10, we want to know uniform annual maintenance cost.
- Or to compute an equivalent A for the maintenance costs to be experienced.
- Solution:
  The equivalent uniform annual maintenance cost is:
- A = 155 + 35(A/G,6%,8yrs) = 155 + 35(3.195)
  = $266.83

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Example 11

- Standard compound interest tables give values for eight interest factors:
  - two single payments,
  - four uniform-payment series, and
  - two uniform gradients.
- These tables do not give the Uniform Gradient Future Worth factor, ***(F/G,i,n).***

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Example 11

- It can be computed from the given tabulated factors:

$$(F/G, i, n) = \frac{(F/A, i, n) - n}{i} = (F/A, i,n)\ x\ (A/G,\ i, n)$$

If i = 10%, n = 12 years,
then (***F/G***,10%,12yrs) =
= (***F/A*** 10%,12yrs) x (***A/G***,10%,12yrs)
= 21.384 x 4.388 = 93.833

Continuous Compounding

Single Payment

Factor	To Find	Given	Function Notation	Formula
Compound Amount	F	P	[F/P, r%, n]	$F = P[e^{rn}]$
Present Worth	P	F	[P/F, r%, n]	$P = F[e^{-rn}]$

Uniform Payment Series

Factor	To Find	Given	Function Notation	Formula
Sinking Fund	A	F	[A/F, r%, n]	$A = F\left[\frac{e^r - 1}{e^{rn} - 1}\right]$
Capital Recovery	A	P	[A/P, r%, n]	$A = P\left[\frac{e^r - 1}{1 - e^{-rn}}\right]$
Compound Amount	F	A	[F/A, r%, n]	$F = A\left[\frac{e^{rn} - 1}{e^r - 1}\right]$
Present Worth	P	A	[P/A, r%, n]	$P = A\left[\frac{1 - e^{-rn}}{e^r - 1}\right]$

Example 12

- $500 is deposited each year into a savings bank account that pays 5% nominal interest, compounded continuously.
- How much will be in the account at the end of five years?

Example 12

- Solution:
  A = \$500, r = 0.05, n = 5 years

$$F = A[F/A, r, n] = A\left[\frac{e^{rn} - 1}{e^{r} - 1}\right]$$

$$F = 500\left[\frac{e^{0.05(5)} - 1}{e^{0.05} - 1}\right] = \$2{,}769.84$$

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COMPOUND INTEREST FACTORS - CONTINUOUS COMPOUNDING

NOMINAL INTEREST RATE = 5.00 PERCENT

N	SINGLE-PAYMENT COMPOUND-AMOUNT FACTOR (F/P)	UNIFORM-SERIES COMPOUND-AMOUNT FACTOR (F/A)	UNIFORM-SERIES CAPITAL-RECOVERY FACTOR (A/P)	GRADIENT SERIES FACTOR (A/G)
1	1.0513	1.0000	1.05127	.0000
2	1.1052	2.0513	.53877	.4875
3	1.1618	3.1564	.36808	.9667
4	1.2214	4.3183	.28284	1.4375
5	1.2840	5.5397	.23179	1.9001
6	1.3499	6.8237	.19782	2.3544
7	1.4191	8.1736	.17362	2.8004
8	1.4918	9.5926	.15552	3.2382
9	1.5683	11.0845	.14149	3.6678
10	1.6487	12.6528	.13031	4.0892
11	1.7333	14.3015	.12119	4.5025
12	1.8221	16.0347	.11364	4.9077
13	1.9155	17.8569	.10727	5.3049
14	2.0138	19.7724	.10185	5.6941
15	2.1170	21.7862	.09717	6.0753
16	2.2255	23.9032	.09311	6.4487
17	2.3396	26.1287	.08954	6.8143
18	2.4596	28.4683	.08640	7.1720
19	2.5857	30.9279	.08360	7.5221
20	2.7183	33.5137	.08111	7.8646
21	2.8577	36.2319	.07887	8.1996
22	3.0042	39.0896	.07685	8.5270
23	3.1582	42.0938	.07503	8.8471
24	3.3201	45.2519	.07337	9.1599
25	3.4903	48.5721	.07186	9.4654
26	3.6693	52.0624	.07048	9.7638
27	3.8574	55.7317	.06921	10.0551
28	4.0552	59.5891	.06805	10.3395
29	4.2631	63.6443	.06698	10.6170
30	4.4817	67.9074	.06600	10.8877
31	4.7115	72.3891	.06509	11.1517

Engineering Economics Problems

- The techniques presented illustrate how to convert single amounts of money, and uniform or gradient series of money, into some equivalent sum at another point in time.
- Compound interest computations are an essential part of engineering economics problems and understanding the Time-Value of Money.

Engineering Economics Problems

- The typical situation is that there are a number of alternatives;
- Which alternative should be selected?
- The customary method of solution is to express each alternative in a common form
- Then choose the best alternative by either maximizing benefits or minimizing costs

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Comparing Alternatives

- There are five major methods:
  1. Present Worth
  2. Future Worth
  3. Annual Cost
  4. Rate-of-Return
  5. Benefit-Cost Analysis

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Present Worth

- Present Worth Analysis converts all of the money consequences of an alternative into an Equivalent Present Sum.

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Present Worth

- Present Worth Analysis is most frequently used to determine the present value of future money receipts and disbursements.
- We might want to know the present value of an income producing property, like an oil well.
- This should provide us with an estimate of the price at which the property could be bought or sold.

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Present Worth

- An important restriction in the use of present worth calculations is that there must be a common analysis period when comparing alternatives.
- It would be incorrect to compare the
  - the present worth of Pump A, expected to last 6 years,
  - with the present worth of Pump B, expected to last 12 years.

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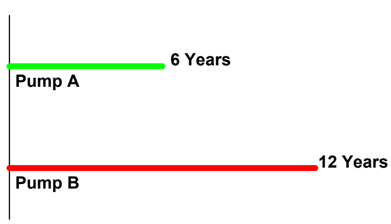
Present Worth Comparison



Improper Present Worth Comparison

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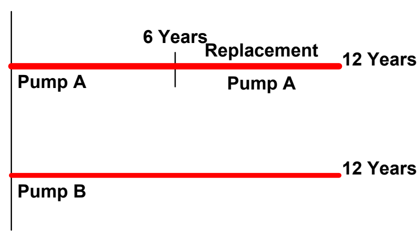
Present Worth Comparison

- A customary assumption would be:
  - that a pump is needed for 12 years and
  - that Pump A will be replaced by an identical Pump A at the end of 6-years.
- This then yields a 12-year analysis period.

Present Worth Comparison



Correct Use of Present Worth Method

Example 13

- Machine X has an initial cost of $10,000, an annual maintenance of $500 per year, and no salvage life at the end of its four-year life.
- Machine Y costs $20,000, and the first year there is no maintenance cost.
- For the second year, maintenance for machine Y is $100, and it increases $100 per year thereafter.

Example 13

- Machine Y has an anticipated $5,000 salvage value at the end of its 12-year useful life.
- If minimum attractive rate of return (MARR) is 8%, which machine should be selected?

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Example 13

- Solution:
- The analysis period was not stated in the problem.
- Therefore, the least common multiple of the lives can be selected,
- or 12 years, as the analysis period.

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Example 13

- PW of Cost of 12 years of Machine X
  PW_X = -10,000 -10,000(P/F,8%,4) + -10,000(P/F,8%,8) - 500(P/A,8%,12) =
- PW_X = -10,000 - 10,000(0.7350) - 10,000(0.5403) - 500(7.536) = -$26,521

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Example 13

- PW of Cost of 12 years of Machine Y
  PW_Y = -20,000 - 100(P/G,8%,12) + 5,000(P/F,8%,12)
- PW_Y= -20,000 -100(34.634) + 5,000(0.3971)
  = -$21,478 (versus PW_x= -$26,521)
- Minimize the PW of Cost
  - PW_Y cost less than PW_X
  - Chose Machine Y.

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Example 14

- Two alternatives have the following cash flows:

Year	Alternative A	Alternative B
0	-$2,000	-$2,800
1	+$800	+$1,100
2	+$800	+$1,100
3	+$800	+$1,100

- At 4% interest rate, which alternative should be selected?

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Example 14

- Compute the NPW of each alternative.
- Net Present Worth (NPW) =
  PW of Benefits – PW of Costs
- NPW_A = 800(P/A,0.04,3) – 2,000 =
  = 800(2.775) – 2,000 =
  $220.00
- NPW_B = 1100(P/A,0.04,3) – 2,800 =
  =1,100(2.775) –2,800 =
  $252.50
- NPW_B > NPW_A,
- Choose Alternative B to Optimize NPW.

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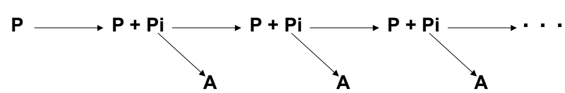
Indefinite Life & Capitalized Cost

- In the special situation where the analysis period is indefinite ($n = \infty$),
- an analysis of the present worth of cost is called capitalized cost.
- There are a few public projects where the analysis is infinity.
- Other examples are permanent endowments and cemetery perpetual care.

Indefinite Life & Capitalized Cost

- When n equals infinity, a present sum P will accrue interest of Pi for every future interest period.
- For the principal sum P to continue undiminished (an essential requirements for n equal to infinity), the end-of-period sum A that can be disbursed is Pi.



Indefinite Life & Capitalized Cost

- With $n = \infty$,
- the fundamental relationship is:
- $A = Pi$
- Some form of this equation is used whenever there is a problem with an infinite analysis period.
- Thus

$$\text{Capital Cost } P = \frac{A}{i}$$

Future Worth Or Value

- In present-worth analysis,
  - the comparison is made in terms of the equivalent present costs and benefits.
- But the analysis doesn't have to be made in terms of present
  - it can be made in terms of a past, present, or future time.
- Although the numerical calculations may look strange, the decision process is unaffected by the selected point in time.

Example 16

- Two alternatives have the following cash flows:

Year	Alternative A	Alternative B
0	-$2,000	-$2,800
1	+$800	+$1,100
2	+$800	+$1,100
3	+$800	+$1,100

At 4% interest rate,
which alternative should be selected?

Example 16

- Solution:
  In Example 14, this problems was solved by Present-Worth analysis at Year 0.
- Here it will be solved by Future-Worth analysis at Year 3.
- Net Future Worth (NFW)
  = FW of Benefits – FW of Costs

Review Comparing Alternatives

- There are five major methods:
  1. Present Worth
  2. Future Worth
  3. Annual Cost
  4. Rate-of-Return
  5. Benefit-Cost Analysis

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Annual Cost

- The annual cost method is more accurately described as the method of Equivalent Uniform Annual Cost (EUAC).
- Or where the computation of benefits, it is called the method of Equivalent Uniform Annual Benefits (EUAB).

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Annual Cost Analysis

- In the present worth method, a common analysis period was required for all alternatives.
- Although this is not required for the Annual Cost Method, it is important to understand the circumstances that justify comparing alternatives with different service lives.

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Annual Cost Analysis

- Frequently this method is used for a more-or-less continuing requirement.
- Pumping water from a well is a good example of requirements on a continuing basis.
- Regardless of whether the pump has a service life of 6 years or 12 years, the minimum annual cost should be used to make the selection.

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Rate Of Return Analysis

- The typical situation is a cash flow representing the costs and benefits.
- The rate of return may be defined as the interest rate where
  - PW of Cost = PW of Benefits, or
  - EUAC = EUAB, or
  - FW of Cost = FW of Benefits.
- The minimum attractive rate of return (MARR) is the smallest interest rate of return at which one is willing to invest money.

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ROR Between Two Alternatives

- Compute the incremental rate of return on cash flow representing the differences between two alternatives.
- Since we want to look at increments of investments, the cash flow for the difference between the alternatives is computed by taking the higher initial-cost alternative minus the lower initial-cost alternative.

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ROR Between Two Alternatives

- If the incremental rate of return is greater than or equal to the predetermined minimum attractive rate of return (MARR),
- Choose the higher-cost alternative;
- Otherwise, choose the lower-cost alternative.

Example 18

- Two alternatives have the cash flows:

Year	Alternative A	Alternative B
0	-$2,000	-$2,800
1	+$800	+$1,100
2	+$800	+$1,100
3	+$800	+$1,100

- If 4% is considered the minimum attractive rate of return (MARR), which alternative should be selected?

Example 18

- Solution:
  These two alternatives were previously examined in Example 14 and 16 by Present-Worth and Future-Worth analysis.
- This time, the alternatives will be resolved using a rate-of-return analysis.

Example 18

- First, tabulate the cash flow that represents the increment of investment between the alternatives.
- This is done by taking the higher initial-cost alternative minus the lower initial-cost alternative.

Year	Alternative A	Alternative B	B-A
0	-$2,000	-$2,800	-$800
1	+800	+1100	+300
2	+800	+1100	+300
3	+800	+1100	+300

Example 18

- Then compute the rate of return on the increment of investment represented by the difference between the alternatives.
- PW of Cost = PW of Benefits

$$800 = 300(P/A,i,3)$$
$$(P/A,i,3) = 800/300 = 2.67$$
$$i = 6.1\%$$

Factor Table - *i* = 6.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9434	0.9434	0.0000	1.0600	1.0000	1.0600	1.0000	0.0000
2	0.8900	1.8334	0.8900	1.1236	2.0600	0.5454	0.4854	0.4854
3	0.8396	2.6730	2.5692	1.1910	3.1836	0.3741	0.3141	0.9612
4	0.7921	3.4651	4.9455	1.2625	4.3746	0.2886	0.2286	1.4272
5	**0.7473**	**4.2124**	**7.9345**	**1.3382**	**5.6371**	**0.2374**	**0.1774**	**1.8836**
6	0.7050	4.9173	11.4594	1.4185	6.9753	0.2034	0.1434	2.3304
7	0.6651	5.5824	15.4497	1.5036	8.3938	0.1791	0.1191	2.7676
8	0.6274	6.2098	19.8416	1.5938	9.8975	0.1610	0.1010	3.1952
9	0.5919	6.8017	24.5768	1.6895	11.4913	0.1470	0.0870	3.6133
10	**0.5584**	**7.3601**	**29.6023**	**1.7908**	**13.1808**	**0.1359**	**0.0759**	**4.0220**
11	0.5268	7.8869	34.8702	1.8983	14.9716	0.1268	0.0668	4.4213
12	0.4970	8.3838	40.3369	2.0122	16.8699	0.1193	0.0593	4.8113
13	0.4688	8.8527	45.9629	2.1329	18.8821	0.1130	0.0530	5.1920
14	0.4423	9.2950	51.7128	2.2609	21.0151	0.1076	0.0476	5.5635
15	**0.4173**	**9.7122**	**57.5546**	**2.3966**	**23.2760**	**0.1030**	**0.0430**	**5.9260**
16	0.3936	10.1059	63.4592	2.5404	25.6725	0.0990	0.0390	6.2794
17	0.3714	10.4773	69.4011	2.6928	28.2129	0.0954	0.0354	6.6240
18	0.3505	10.8276	75.3569	2.8543	30.9057	0.0924	0.0324	6.9597
19	0.3305	11.1581	81.3062	3.0256	33.7600	0.0896	0.0296	7.2867
20	**0.3118**	**11.4699**	**87.2304**	**3.2071**	**36.7856**	**0.0872**	**0.0272**	**7.6051**
21	0.2942	11.7641	93.1136	3.3996	39.9927	0.0850	0.0250	7.9151
22	0.2775	12.0416	98.9412	3.6035	43.3923	0.0830	0.0230	8.2166
23	0.2618	12.3034	104.7007	3.8197	46.9958	0.0813	0.0213	8.5099
24	0.2470	12.5504	110.3812	4.0489	50.8156	0.0797	0.0197	8.7951
25	**0.2330**	**12.7834**	**115.9732**	**4.2919**	**54.8645**	**0.0782**	**0.0182**	**9.0722**
30	0.1741	13.7648	142.3588	5.7435	79.0582	0.0726	0.0126	10.3422
40	0.0972	15.0463	185.9568	10.2857	154.7620	0.0665	0.0065	12.3590
50	0.0543	15.7619	217.4574	18.4202	290.3359	0.0634	0.0034	13.7964
60	0.0303	16.1614	239.0428	32.9877	533.1282	0.0619	0.0019	14.7909
100	**0.0029**	**16.6175**	**272.0471**	**339.3021**	**5,638.3681**	**0.0602**	**0.0002**	**16.3711**

Example 18

- Since the incremental rate of return exceeds the 4% MARR, the increment of investment is desirable.
- Choose the higher-cost, Alternative B.
- Before leaving this example, note the relationship of the rates of return on Alternative A and on Alternative B.

Example 18

- These rates of return are:

	Rate of Return
Alternative A	9.7%
Alternative B	8.7%

- The correct answer to this problem has been shown to be Alternative B, even though Alternative A has a higher rate of return.

Example 18

- Two alternatives have the cash flows:

Year	Alternative A	Alternative B
0	-$2,000	-$2,800
1	+$800	+$1,100
2	+$800	+$1,100
3	+$800	+$1,100

- If 4% is considered the minimum attractive rate of return (MARR), which alternative should be selected?

Review Comparing Alternatives

- There are five major methods:
  1. Present Worth
  2. Future Worth
  3. Annual Cost
  4. Rate-of-Return
  5. Benefit-Cost Analysis

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Benefit-Cost Analysis

- Generally, in public works and governmental economic analyses, the dominant method of analysis is the Benefit-Cost Ratio (B/C).
- It is simply the ratio of benefits divided by costs, taking into account the time value of money.

$$\text{B/C Ratio} = \frac{\text{PW}_{\text{Benefits}}}{\text{PW}_{\text{Costs}}} = \frac{\text{FW}_{\text{Benefits}}}{\text{FW}_{\text{Costs}}} = \frac{\text{EUAB}}{\text{EUAC}}$$

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Benefit-Cost Analysis

- For a given interest rate,
  a B/C Ratio ≥ 1 or B – C ≥ 1
  reflects an acceptable project.
- The B/C analysis method is parallel to that of rate-of-return analysis.
- The same kind of incremental analysis is required when comparing two alternatives.

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Example 20

- With i=10%, Solve Example 20 by Benefit-Cost Analysis.

Year	Alternative A	Alternative B	A – B
0	-$200.0	-$131.0	-$69.0
1	+77.6	+48.1	+29.5
2	+77.6	+48.1	+29.5
3	+77.6	+48.1	+29.5

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Example 20

- The benefit-cost ratio for A – B increment is

$$B/C = \frac{\text{PW of Benefits}}{\text{PW of Costs}} = \frac{29.5(P/A, i, n)}{69.0} = \frac{73.37}{69.0} = 1.06$$

  - Since the B/C ratio exceeds 1, the increment of investment is desirable.
  - Select the higher cost Alternative A.

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Valuation And Depreciation

- Depreciation is the systematic allocation of the cost of a capital asset over its useful life.
- Book value is the original cost of an asset (C), minus the accumulated depreciation of the asset (Σ(Dj)).
- Book Value (BV) = C - Σ(Dj)

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Valuation & Depreciation Notation

- BV = Book Value
- C = Cost of Property (Basis)
- Dj = Depreciation in Year j
- Sn = Salvage Value in Year n

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Items to Consider

1. Cost of the Property, C (called the basis in tax law).
2. Type of property. Property is classified either as
   - tangible (like machinery) or
   - intangible (like a franchise or a copyright) and
   - either real property (real estate) or
   - personal property (everything not real property).

Items to Consider

3. Depreciable Life in years, n.
4. Salvage Value of the property at the end of its depreciable (usable) life, Sn.

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Straight-Line Depreciation

- Depreciation Charge in any year, the Cost of Property minus the Salvage Value divided by the Number of Years of Useful Life.

$$Dj = \frac{C - Sn}{n}$$

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Straight Line Depreciation

- You buy a styling new car for $15,000 with an anticipated salvage value of $1,500 at the end of its five-year depreciation life.
- Compute the depreciation schedule for the car by the straight line method.

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Modified Accelerated Cost Recovery System Depreciation

- The Modified-Accelerated-Cost-Recovery-System (MACRS) depreciation method .
  - D_j = C x factor

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MACRS Depreciation Factors

Modified ACRS (MACRS)
Depreciation for Personal Property – Half-Year Convention

Recovery year is:	3-Year recovery	5 –Year recovery	7-Year recovery	10-Year recovery
1	33.3	20.0	14.3	10.0
2	44.5	32.0	24.5	18.0
3	14.8	19.2	17.5	14.4
4	7.4	11.5	12.5	11.5
5		11.5	8.9	9.2
6		5.8	8.9	7.4
7			8.9	6.6
8			4.5	6.6
9				6.5
10				6.5
11				3.3

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MACRS Depreciation

- To compute the MACRS depreciation for an item you need to know:

1. Cost (basis) of the item.
2. Property Class
   - All tangible property is classified in one of six classes (3, 5, 7, 10, 15, and 20 years), which is the life over which it is depreciated
   - Residential real estate and nonresidential real estate are in two separate real property classes of 27.5 years and 39 years, respectively.
4. In MACRS, the salvage value is assumed to be zero

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MACRS: Half-Year Convention

- Except for real property, a half-year convention is used.
- Under this convention all property is considered to be placed in service in the middle of the tax year, and a half-year of depreciation is allowed in the first year.
- For each of the remaining years, one is allowed a full year of depreciation.

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MACRS: Half-Year Convention

- If the property is disposed of prior to the end of the recovery period (property class life), a half year depreciation is allowed in that year.
- If the property is held for the entire recovery period, a half-year of depreciation is allowed following the end of the recovery period.

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MACRS Depreciation Factors

Modified ACRS (MACRS)
Depreciation for Personal Property – Half-Year Convention

Recovery year is:	3-Year recovery	5 –Year recovery	7-Year recovery	10-Year recovery
1	33.3	20.0	14.3	10.0
2	44.5	32.0	24.5	18.0
3	14.8	19.2	17.5	14.4
4	7.4	11.5	12.5	11.5
5		11.5	8.9	9.2
6		5.8	8.9	7.4
7			8.9	6.6
8			4.5	6.6
9				6.5
10				6.5
11				3.3

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Example 21

- A $5,000 GPS unit has an anticipated $500 salvage value at the end of its five-year depreciation life.
- Compute the depreciation schedule for the machinery by MACRS depreciation.
- Do the MACRS computation, and compare the results with the values from a straight line depreciation.

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Example 21

- Solution:
  straight line depreciation.

  Dj=(5000-500)/5 = 900

Year	Dj	BV @ end of the year
1	900	5000-900=4100
2	900	4100-900=3200
3	900	2300
4	900	1400
5	900	500

Example 21

- MACRS Solution:
  - Five-year property class.
  - Salvage value Sn is assumed to be zero for MACRS.
  - Use depreciation factors from MACRS Table

Year	Factor	Dj	BV @ half year
1	20.0%	5000*.20 = 1000	5000-1000 = 4000
2	32.0%	5000*.32 = 1600	4000-1600=2400
3	19.2%	5000*.192 =960	2400-960=1440
4	11.5%	5000*.115 = 575	1440-575=865
5	11.5%	5000*.115 = 575	865-575=290
6	5.8%	5000*.058 = 290	290-290=0

MACRS Depreciation Factors

Modified ACRS (MACRS)
Depreciation for Personal Property – Half-Year Convention

Recovery year is:	3-Year recovery	5 –Year recovery	7-Year recovery	10-Year recovery
1	33.3	20.0	14.3	10.0
2	44.5	32.0	24.5	18.0
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4	7.4	11.5	12.5	11.5
5		11.5	8.9	9.2
6		5.8	8.9	7.4
7			8.9	6.6
8			4.5	6.6
9				6.5
10				6.5
11				3.3

Example 21

Straight Line

Year	BV @ end of the year
1	4100
2	3200
3	2300
4	1400
5	500

VS

MACRS

Year	BV @ half year
1	4000
2	2400
3	1440
4	865
5	290
6	0

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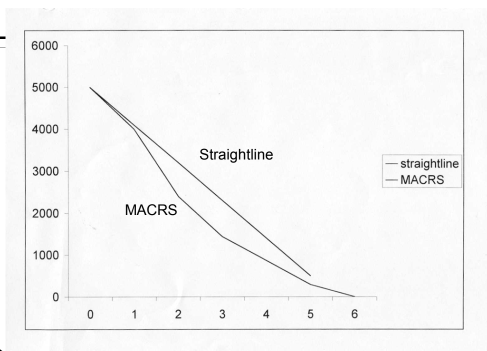

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Inflation

- Inflation is characterized by rising prices for goods and services,
- Deflation produces a fall in prices for goods and services.

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Inflation

- An inflationary trend makes future dollars have less purchasing power than present dollars.
- This helps long-term borrowers for they repay a loan of present dollars in the future with dollars of reduced buying power.
- The help to borrowers is at the expense of lenders.

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Inflation

- Price changes occur in a variety of ways.
- One method of stating a price change is as a uniform rate of price change per year.
- Notation: f = General Inflation Rate per Interest Period

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Example 22

- One economist has predicted that there will be a 7% per year inflation of prices during the next ten years.
- If this proves to be correct, an item that presently sells for $10 would sell for what price ten years hence?

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Example 22

- Solution:
  f = 7%, P = \$10, F = ?, n = 10 years
- Here the computation is find the future worth F, rather than the present worth P.
- $F = P(1+f)^{10}$ = $10(1+0.07)^{10}$ = \$19.67

Example 23

- A mortgage will be repaid in three equal payments of \$5,000 at the end of Year 1, 2, and 3.
- If the annual inflation rate, f, is 8% during this period, and
- the investor wants a 12% annual interest rate (i),
- what is the maximum amount he would be willing to pay for the mortgage?

Example 23

- The computation is a two-step process.
- First, the three future payments must be converted into dollars with the same purchasing power as today's (Year 0) dollars.

Year	Actual Cash Flow		Multiplied By		Cash flow adjusted To today's (yr. 0) dollars
0					
1	+5000	X	$(1+0.08)^{-1}$	=	+4630
2	+5000	X	$(1+0.08)^{-2}$	=	+4286
3	+5000	X	$(1+0.08)^{-3}$	=	+3969

Example 23

- The general form of the adjusting multiplier is $(1+f)^{-n}$ which equals ***(P/F,f,n)***.
- Now that the problem has been converted to dollars of the same purchasing power (today's dollars in this example), we can proceed to compute the present worth of the future payments.

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Example 23

Year	Actual Cash Flow		Multiplied By		Cash flow adjusted To today's (yr. 0) dollars
0					
1	+4630	X	$(1+0.12)^{-1}$	=	+4134
2	+4286	X	$(1+0.12)^{-2}$	=	+3417
3	+3969	X	$(1+0.12)^{-3}$	=	+2825
					$10,376

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Alternate Solution

- Instead of doing the inflation and interest rate computations separately, you can compute a combined equivalent interest rate per interest period, d.
- $d = (1+f)(1+i) - 1 = i + f + (i \times f)$
- For this cash flow,
  d = 0.12 + 0.08 + 0.12(0.08) = =

  0.2096. or 20.96%

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Alternate Solution

- ***PW*** = 5,000(1+0.2096) $^{-1}$ +
  5,000(1+0.2096) $^{-2}$ +
  5,000(1+0.2096) $^{-3}$ =
- ***PW*** = 4,134 + 3,417 + 2,825 = $10,376

~OR~

- PW = 5000*[(1+0.2096)^3-1]/[(0.2096)(1+0.2096)^3]
- PW=$10,376

Factor Table - *i* = 6.00%

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.9434	0.9434	0.0000	1.0600	1.0000	1.0600	1.0000	0.0000
2	0.8900	1.8334	0.8900	1.1236	2.0600	0.5454	0.4854	0.4854
3	0.8396	2.6730	2.5692	1.1910	3.1836	0.3741	0.3141	0.9612
4	0.7921	3.4651	4.9455	1.2625	4.3746	0.2886	0.2286	1.4272
5	**0.7473**	**4.2124**	**7.9345**	**1.3382**	**5.6371**	**0.2374**	**0.1774**	**1.8836**
6	0.7050	4.9173	11.4594	1.4185	6.9753	0.2034	0.1434	2.3304
7	0.6651	5.5824	15.4497	1.5036	8.3938	0.1791	0.1191	2.7676
8	0.6274	6.2098	19.8416	1.5938	9.8975	0.1610	0.1010	3.1952
9	0.5919	6.8017	24.5768	1.6895	11.4913	0.1470	0.0870	3.6133
10	**0.5584**	**7.3601**	**29.6023**	**1.7908**	**13.1808**	**0.1359**	**0.0759**	**4.0220**
11	0.5268	7.8869	34.8702	1.8983	14.9716	0.1268	0.0668	4.4213
12	0.4970	8.3838	40.3369	2.0122	16.8699	0.1193	0.0593	4.8113
13	0.4688	8.8527	45.9629	2.1329	18.8821	0.1130	0.0530	5.1920
14	0.4423	9.2950	51.7128	2.2609	21.0151	0.1076	0.0476	5.5635
15	**0.4173**	**9.7122**	**57.5546**	**2.3966**	**23.2760**	**0.1030**	**0.0430**	**5.9260**
16	0.3936	10.1059	63.4592	2.5404	25.6725	0.0990	0.0390	6.2794
17	0.3714	10.4773	69.4011	2.6928	28.2129	0.0954	0.0354	6.6240
18	0.3505	10.8276	75.3569	2.8543	30.9057	0.0924	0.0324	6.9597
19	0.3305	11.1581	81.3062	3.0256	33.7600	0.0896	0.0296	7.2867
20	**0.3118**	**11.4699**	**87.2304**	**3.2071**	**36.7856**	**0.0872**	**0.0272**	**7.6051**
21	0.2942	11.7641	93.1136	3.3996	39.9927	0.0850	0.0250	7.9151
22	0.2775	12.0416	98.9412	3.6035	43.3923	0.0830	0.0230	8.2166
23	0.2618	12.3034	104.7007	3.8197	46.9958	0.0813	0.0213	8.5099
24	0.2470	12.5504	110.3812	4.0489	50.8156	0.0797	0.0197	8.7951
25	**0.2330**	**12.7834**	**115.9732**	**4.2919**	**54.8645**	**0.0782**	**0.0182**	**9.0722**
30	0.1741	13.7648	142.3588	5.7435	79.0582	0.0726	0.0126	10.3422
40	0.0972	15.0463	185.9568	10.2857	154.7620	0.0665	0.0065	12.3590
50	0.0543	15.7619	217.4574	18.4202	290.3359	0.0634	0.0034	13.7964
60	0.0303	16.1614	239.0428	32.9877	533.1282	0.0619	0.0019	14.7909
100	**0.0029**	**16.6175**	**272.0471**	**339.3021**	**5,638.3681**	**0.0602**	**0.0002**	**16.3711**

Factor Table - *i* = 12.00%

n	P/F	P/A	P/G	F/P	F/A	A/P	A/F	A/G
1	0.8929	0.8929	0.0000	1.1200	1.0000	1.1200	1.0000	0.0000
2	0.7972	1.6901	0.7972	1.2544	2.1200	0.5917	0.4717	0.4717
3	0.7118	2.4018	2.2208	1.4049	3.3744	0.4163	0.2963	0.9246
4	0.6355	3.0373	4.1273	1.5735	4.7793	0.3292	0.2092	1.3589
5	**0.5674**	**3.6048**	**6.3970**	**1.7623**	**6.3528**	**0.2774**	**0.1574**	**1.7746**
6	0.5066	4.1114	8.9302	1.9738	8.1152	0.2432	0.1232	2.1720
7	0.4523	4.5638	11.6443	2.2107	10.0890	0.2191	0.0991	2.5515
8	0.4039	4.9676	14.4714	2.4760	12.2997	0.2013	0.0813	2.9131
9	0.3606	5.3282	17.3563	2.7731	14.7757	0.1877	0.0677	3.2574
10	**0.3220**	**5.6502**	**20.2541**	**3.1058**	**17.5487**	**0.1770**	**0.0570**	**3.5847**
11	0.2875	5.9377	23.1288	3.4785	20.6546	0.1684	0.0484	3.8953
12	0.2567	6.1944	25.9523	3.8960	24.1331	0.1614	0.0414	4.1897
13	0.2292	6.4235	28.7024	4.3635	28.0291	0.1557	0.0357	4.4683
14	0.2046	6.6282	31.3624	4.8871	32.3926	0.1509	0.0309	4.7317
15	**0.1827**	**6.8109**	**33.9202**	**5.4736**	**37.2797**	**0.1468**	**0.0268**	**4.9803**
16	0.1631	6.9740	36.3670	6.1304	42.7533	0.1434	0.0234	5.2147
17	0.1456	7.1196	38.6973	6.8660	48.8837	0.1405	0.0205	5.4353
18	0.1300	7.2497	40.9080	7.6900	55.7497	0.1379	0.0179	5.6427
19	0.1161	7.3658	42.9979	8.6128	63.4397	0.1358	0.0158	5.8375
20	**0.1037**	**7.4694**	**44.9676**	**9.6463**	**72.0524**	**0.1339**	**0.0139**	**6.0202**
21	0.0926	7.5620	46.8188	10.8038	81.6987	0.1322	0.0122	6.1913
22	0.0826	7.6446	48.5543	12.1003	92.5026	0.1308	0.0108	6.3514
23	0.0738	7.7184	50.1776	13.5523	104.6029	0.1296	0.0096	6.5010
24	0.0659	7.7843	51.6929	15.1786	118.1552	0.1285	0.0085	6.6406
25	**0.0588**	**7.8431**	**53.1046**	**17.0001**	**133.3339**	**0.1275**	**0.0075**	**6.7708**
30	0.0334	8.0552	58.7821	29.9599	241.3327	0.1241	0.0041	7.2974
40	0.0107	8.2438	65.1159	93.0510	767.0914	0.1213	0.0013	7.8988
50	0.0035	8.3045	67.7624	289.0022	2,400.0182	0.1204	0.0004	8.1597
60	0.0011	8.3240	68.8100	897.5969	7,471.6411	0.1201	0.0001	8.2664
100		**8.3332**	**69.4336**	**83,522.2657**	**696,010.5477**	**0.1200**		**8.3321**

Engineering Economics