# *FE/EIT Circuits Review*

## Agenda

- DC Circuits
  - Basic Definitions
  - Circuit Components
  - DC Circuit Laws
  - Kirchoff's Laws
  - Civil Engineering Example

- AC Circuits

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DC Glossary of Terms/Quantities

Q – Charge (+/-) unit - coulomb

electron $\approx -1.6 \times 10^{-19}$ C

I – Current unit - Amperes =

$i(t) = \frac{dQ}{dt}$ (direction) Coulomb/sec

V – Voltage (potential) polarity

unit: Joule/coulomb = Volt

W – Energy (unit = Joules)

P - Power = rate of change of

of Energy $= \frac{dw}{dt} \Rightarrow$ unit

Joules/sec = watts

(power can be delivered

or absorb)

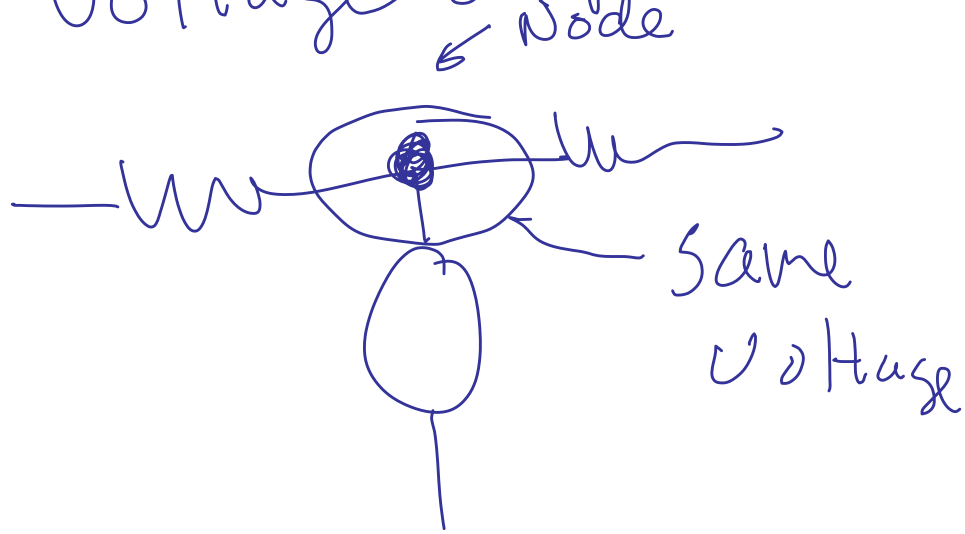
# FE/EIT Circuits Review

## Glossary of Terms/Quantities

Circuit - ~~co~~ network of two or more circuit elements connected in such a way that current can flow.

Node - a junction of two or more circuit elements. we assume that the node is at one voltage or potential

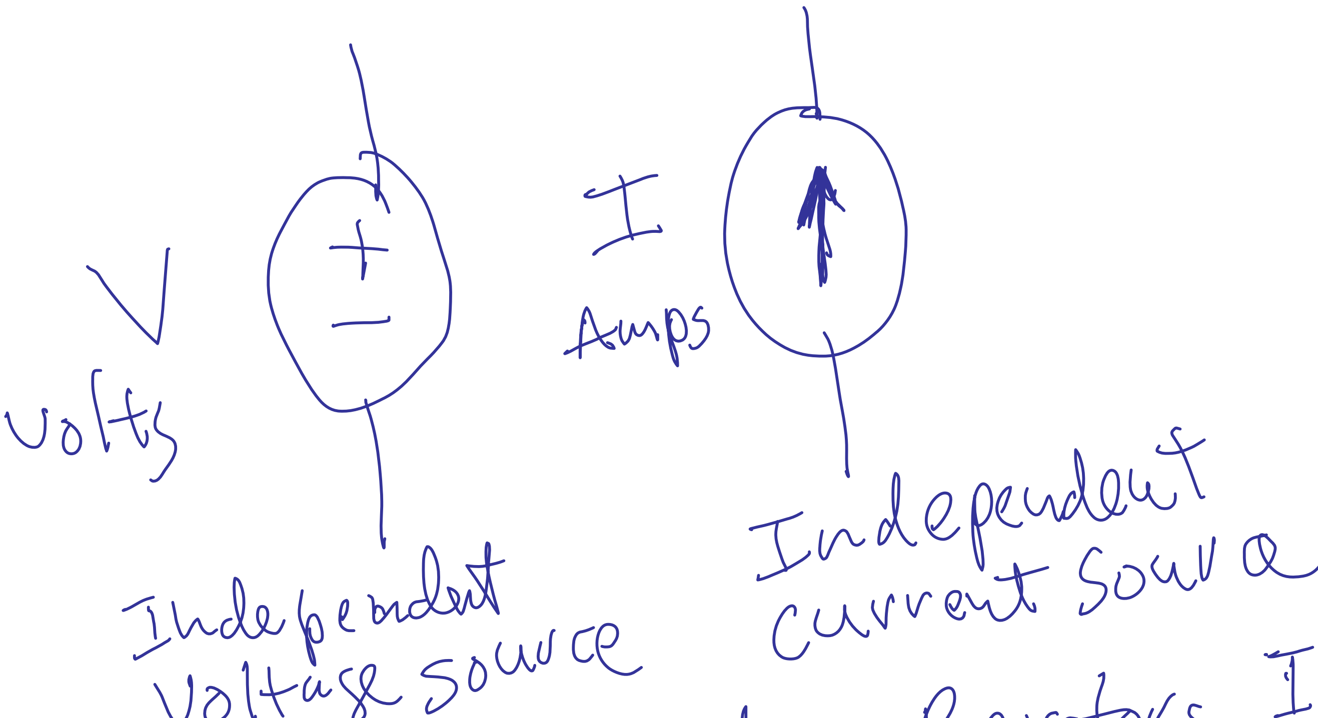
ex.



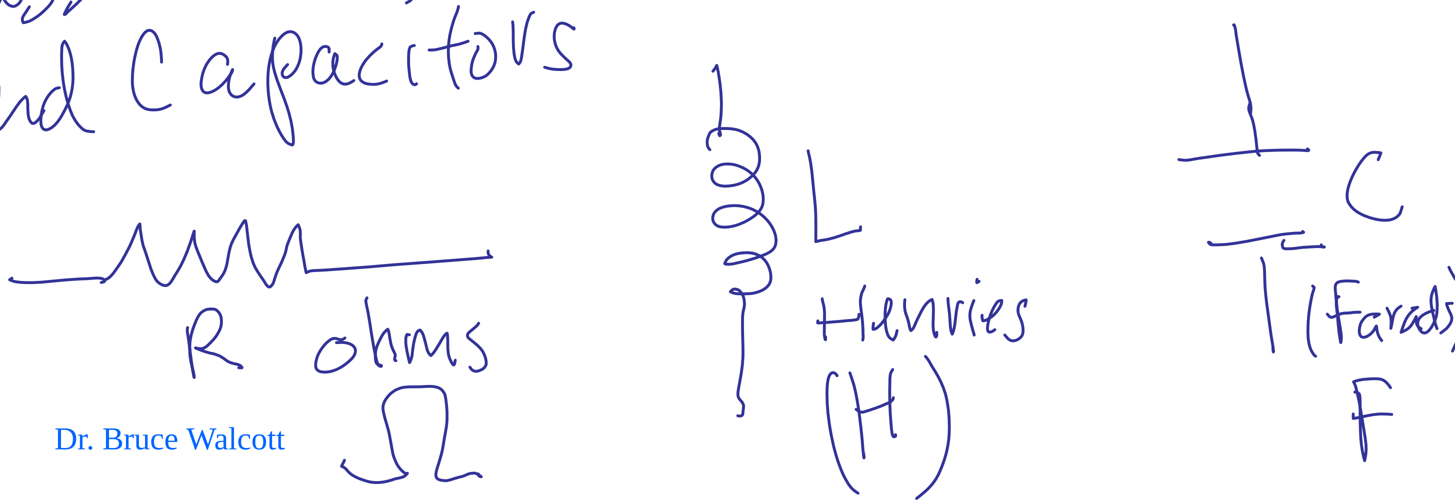
# DC Circuits

## Components:

active components capable of delivering power <u>forever</u> to the rest of the circuit;

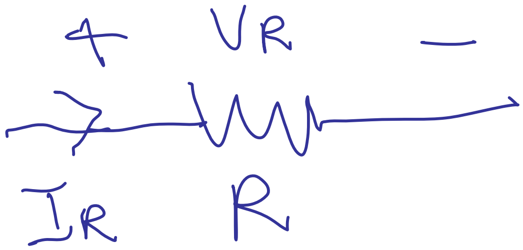



Passive components: Resistors, Inductors and Capacitors



# *DC Circuit Laws*

## Ohm's Law and Passive Sign Convention

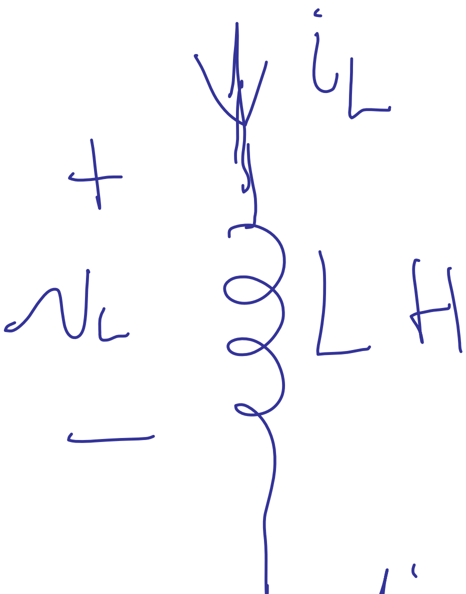


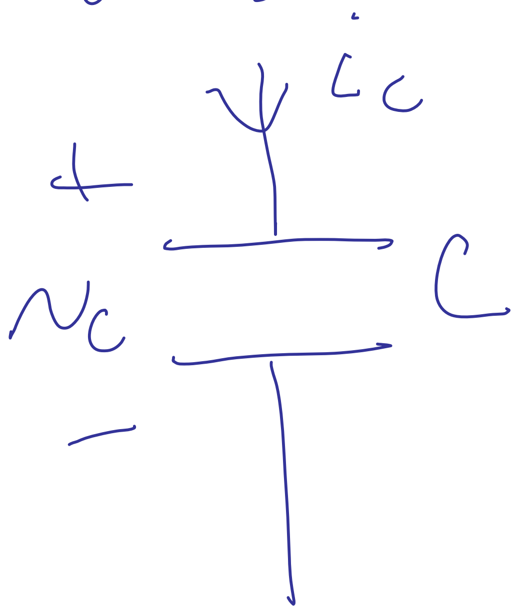

Ohm's Law:

$$\boxed{V = IR}$$

Passive sign Convention – Lauren observed that current must enter (+) terminal for ohm's law to work

## Dual for Inductor and Capacitor




$$v_L = L\frac{di}{dt}$$

$$i_C = C\frac{dv_C}{dt}$$

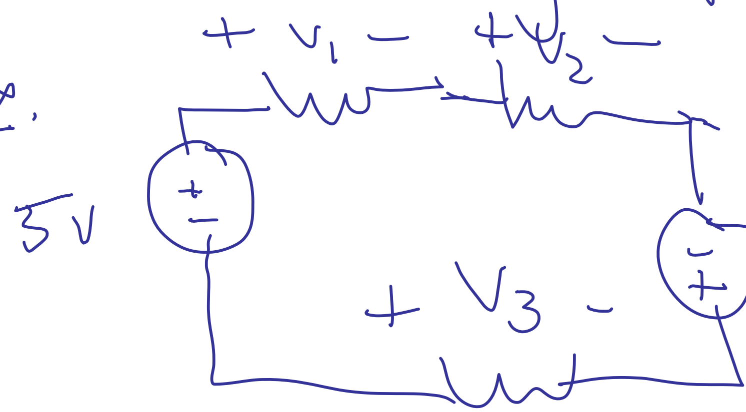
# DC Circuit Laws

## Kirchoff's Laws:

Kirchoff's Voltage Law (KVL):

The algebraic sum of voltages around any loop in a circuit is 0.

Ex.



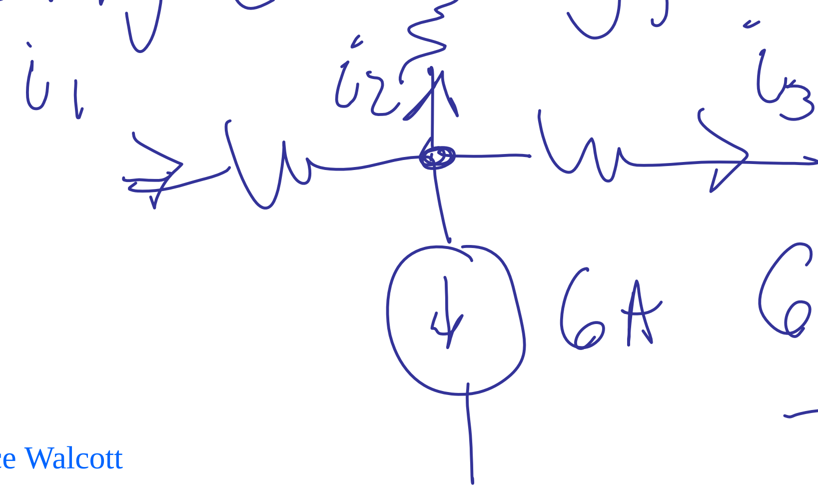

KVL:

$-5 + V_1 + V_2 - 3 - V_3 = 0$

Kirchoff's Current Law (KCL):

The algebraic sum of currents leaving (entering) a node is 0.

Ex.




KCL:

$6A - i_1 + i_2 + i_3 = 0$

# *Civil Engineering Circuit Analogy*

In the fluid system shown,

Pressure is analogous to Voltage.

Resistance of the pipe is analogous to resistance

Flow is analogous to current

# *FE Practice Problems*

*6.1 For the circuit below, with the voltages' polarities as shown, KVL in equation form is

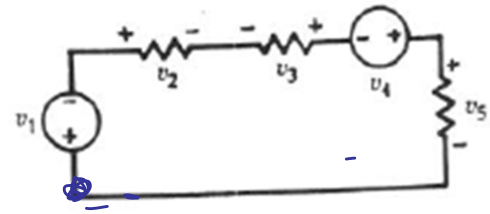
a) $v_1 + v_2 + v_3 - v_4 + v_5 = 0$

b) $-v_1 + v_2 + v_3 - v_4 + v_5 = 0$

c) $v_1 + v_2 - v_3 - v_4 + v_5 = 0$

d) $-v_1 - v_2 - v_3 + v_4 + v_5 = 0$

e) $v_1 - v_2 + v_3 + v_4 - v_5 = 0$



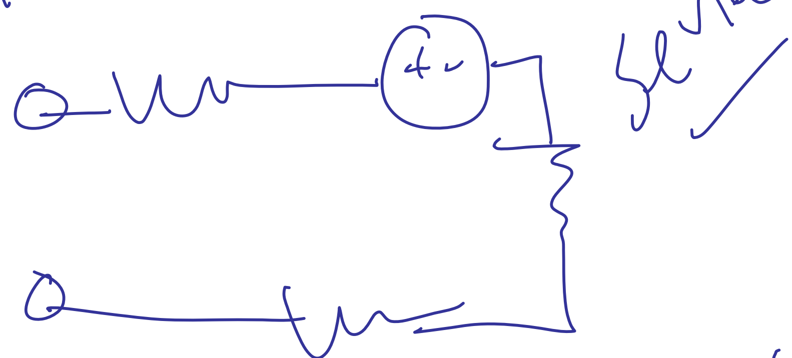

KVL: $+V_1 + V_2 - V_3 - V_4 + V_5 = 0$

# Series and Parallel

Definition of series:

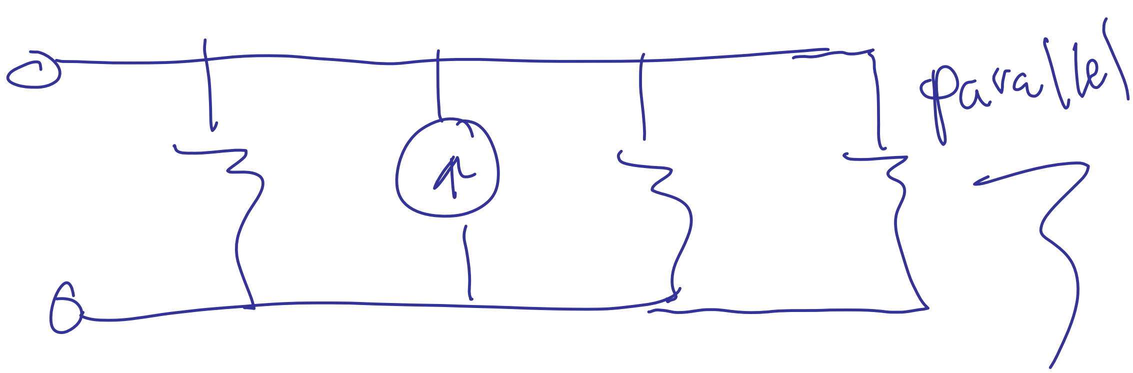
Two or more circuit elements are in series if and only if the same current flows through them

Ex

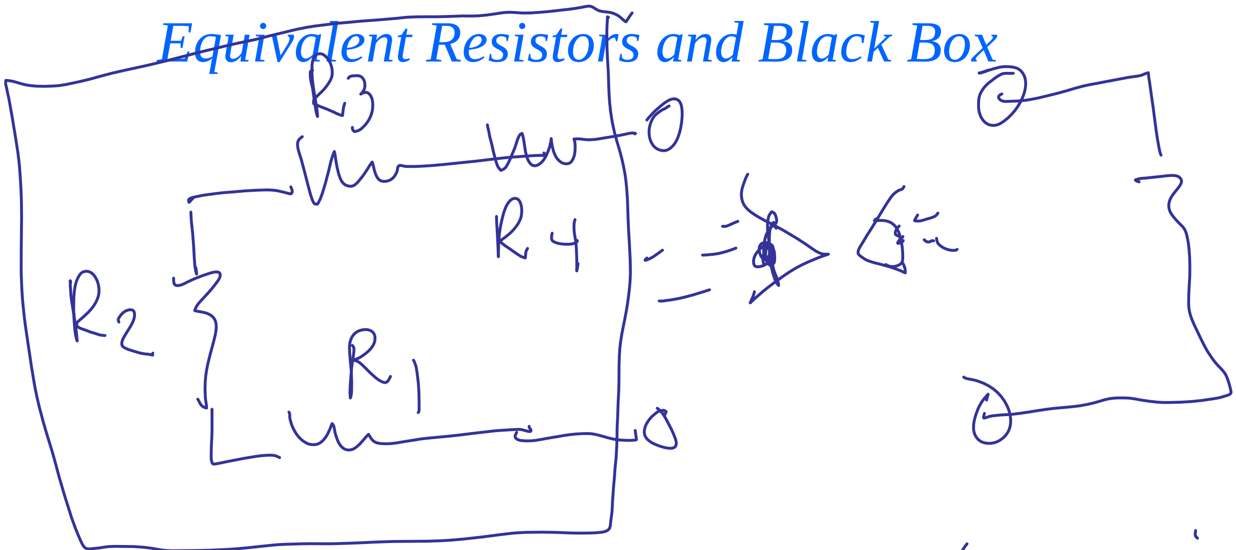



Parallel: Two or more elements are in parallel if and only if the same voltage appears across all of them.

Ex



# Equivalent Resistors and Black Box

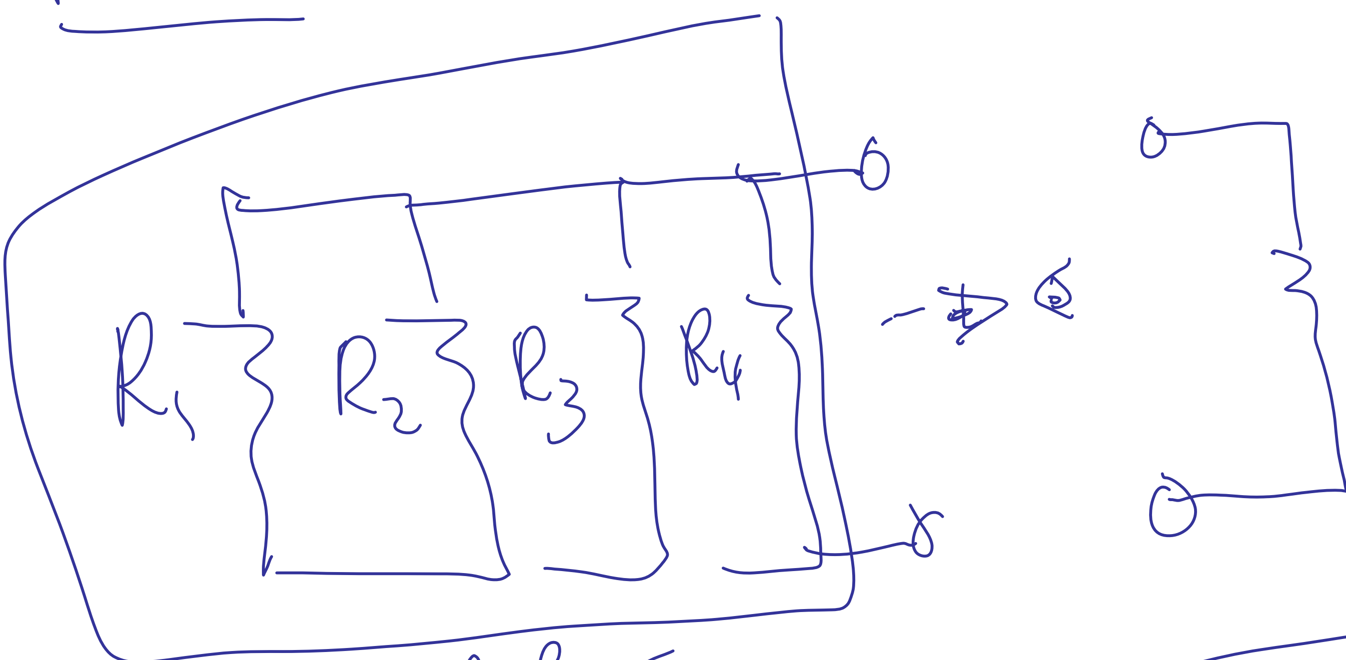



Blue Box

$R_{eq} = R_1 + R_2 + R_3 + R_4$

Resistors in series add

Dual Resistors in Parallel



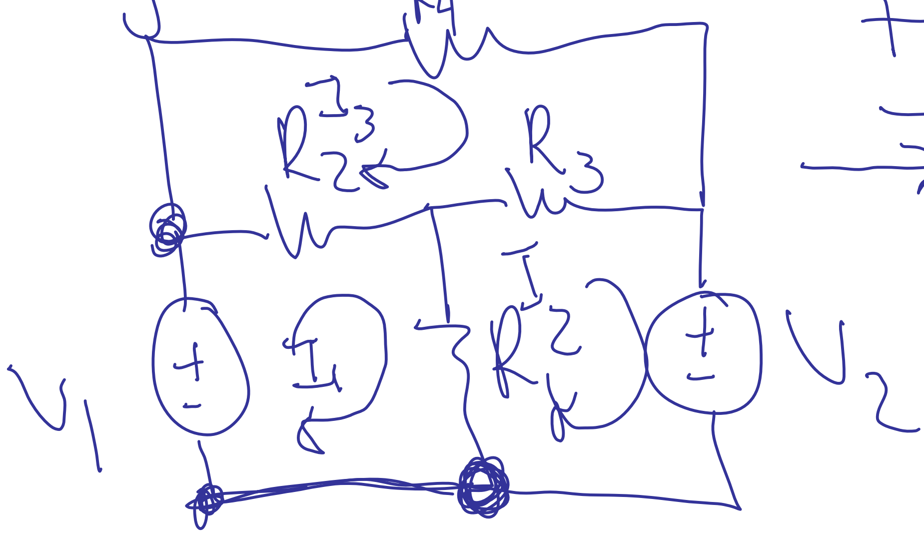

$R_{eq} = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$

$\dfrac{1}{1/R_1 + 1/R_2} = \dfrac{R_1 R_2}{R_1 + R_2}$

# *Mesh Analysis*

Problem: How do I analyze a multi-loop circuit?

Ex





Solution: Assign mesh currents flowing around each individual loop of the circuit. Then write a KVL for each loop treating the mesh currents like ocean currents

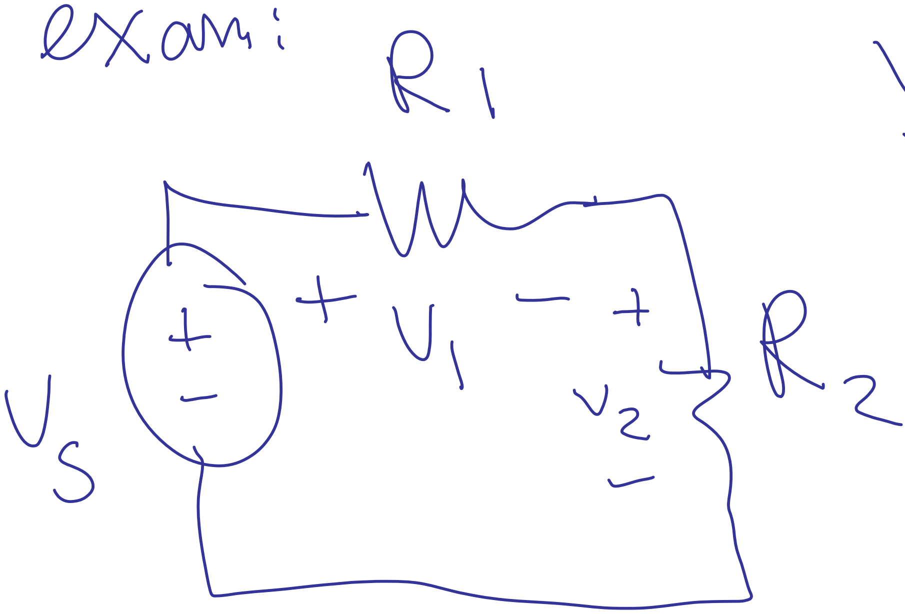
Loop 1: $-V_1 + R_2(I_1 - I_3) + R_1(I_1 - I_2) = 0$

Loop 2: $+R_1(I_2 - I_1) + R_3(I_2 - I_3) + V_2 = 0$

Loop 3: $+R_4(I_3 - 0) + R_3(I_3 - I_2) + R_2(I_3 - I_1) = 0$

# *FE Trick: Voltage and Current Division*

These situations are <u>common</u> on FE exam:

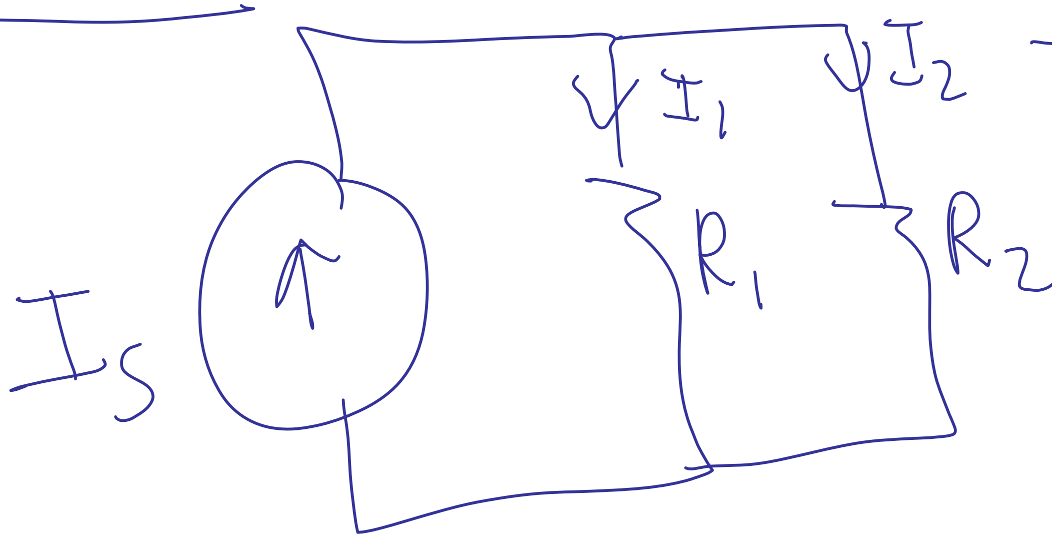



$$\underline{V_1 + V_2 = V_S}$$

$$V_1 = \frac{R_1 V_S}{R_1 + R_2}$$

$$V_2 = \frac{R_2 V_S}{R_1 + R_2}$$

<u>Dual:</u>




$$I_1 = \frac{R_2 I_S}{R_1 + R_2}$$

$$I_2 = \frac{R_1 I_S}{R_1 + R_2}$$

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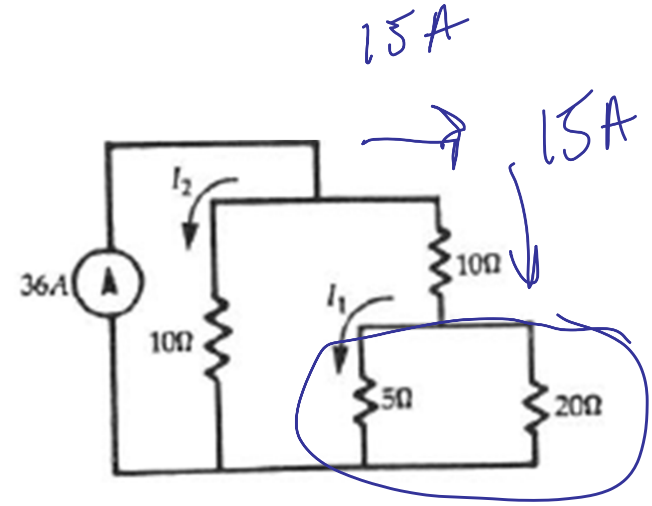
# Practice Problems (cont.)

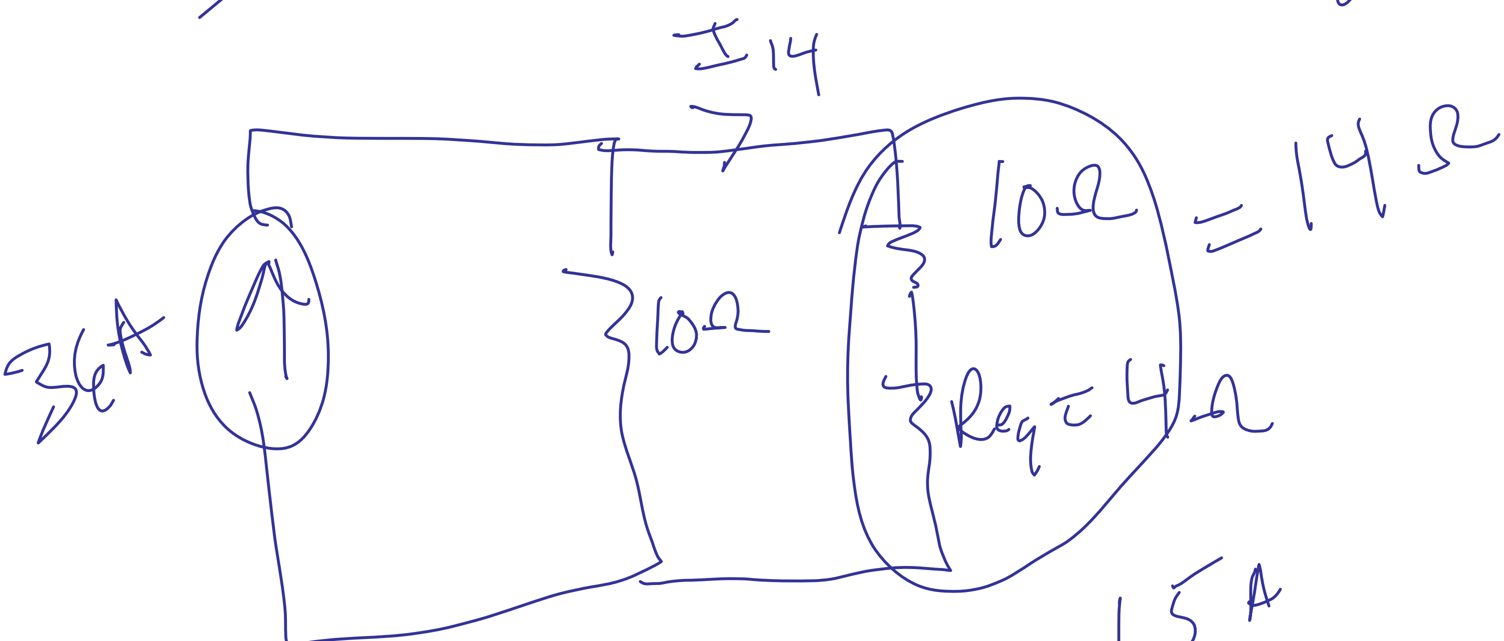
6.2 Find $I_1$ in amps.

a) 12
b) 15
c) 18
d) 21
e) 27




~~KVL mesh~~ analysis

$$R_{eq} = \frac{1}{\frac{1}{5}+\frac{1}{20}} = 4\Omega$$




$$I_{14} = \frac{10}{10+14}\, 36A = 15A$$

By current division again

$$I_1 = \frac{20}{5+20} \cdot 15 = 12A$$

# *Power – WATT is it?*

$$\text{Power} = V \times I$$

$$\text{Power absorbed} = V \times I$$

When V and I satisfy the passive sign convention.

Power absorbed = − Power delivered

Hint: always find power absorbed by an element by satisfying passive sign convention.

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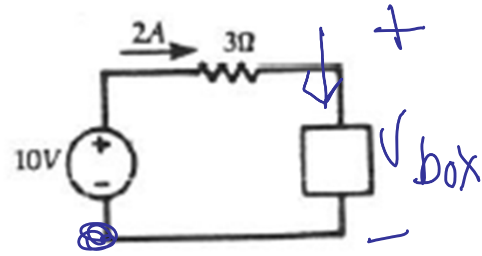
# *Practice Problems (cont.)*

+6v = $I_{box}$

*6.3 Find the magnitude and sign of the power, in watts, absorbed by the circuit element in the box.

a) -20
b) -8
c) 8
d) 12
e) 20

Power delivered = $-8$ W




Power absorbed = $V \times I$ if passive sign convention is met

$I_{box} = 2A$

KVL: $-10 + (2A \times 3\Omega) + V_{box} = 0$

$V_{box} = 4v \Rightarrow$ Power absorbed

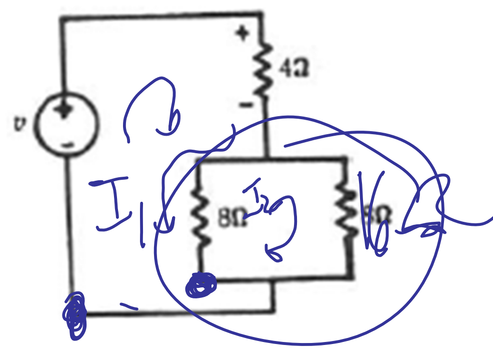
$= V_{box} \times I_{box}$

$4 \times 2 = 8W$

$I_1 = 1/8\,A$

# *Practice Problems (cont.)*

*6.4 For the circuit shown, the voltage across the 4 ohm resistor is, with $v = 1$ V




a) ~~1/4~~
b) 1/2
c) 2/3
d) 2
e) 4

$R_{eq} = 4\Omega$

Mesh Analysis:

$L_1: -V + 4(I_1 - 0) + 8(I_1 - I_2) = 0$

$L_2: +8(I_2 - I_1) + 8(I_2 - 0) = 0$

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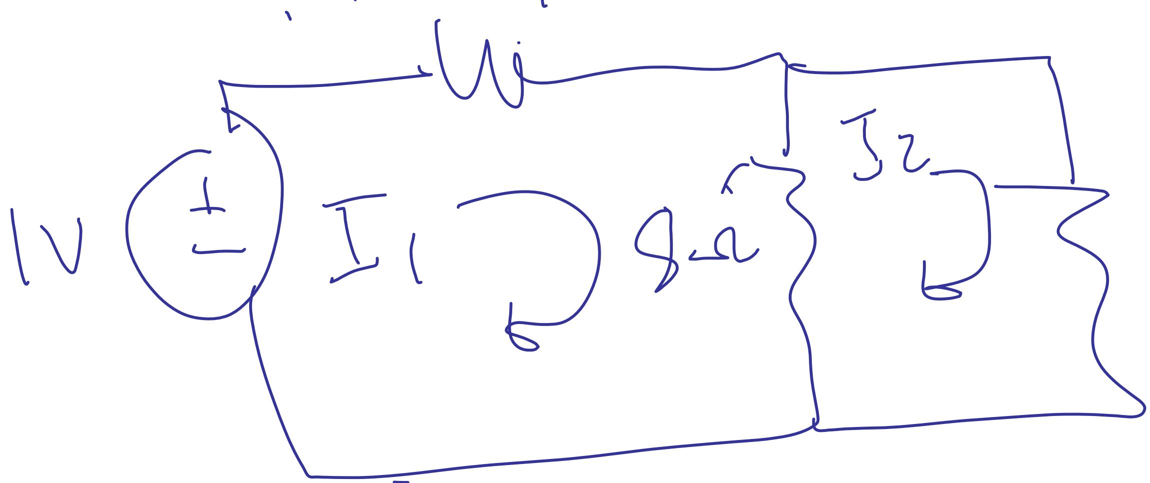
Voltage division $V_4 = \frac{4}{4+4} \cdot 1V = 1/2\,V$

4Ω 4Ω



# Conductance – Dual of Resistance

Conductance is the inverse of Resistance:

Symbol is G and the units are mho's


(mhos)

$$\boxed{G = \frac{1}{R}}$$

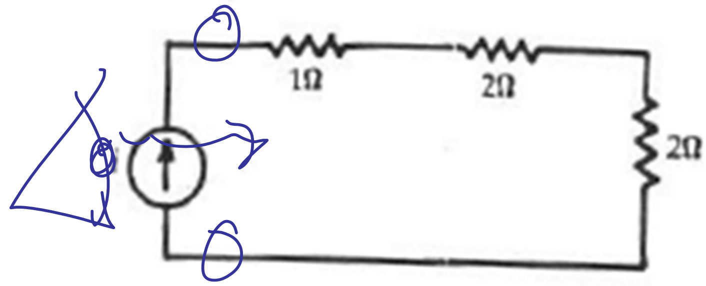
$$I = V \cdot G$$

Combine just like springs

## *Practice Problems (cont.)*

6.5 The total conductance, in mhos, in the circuit shown below is

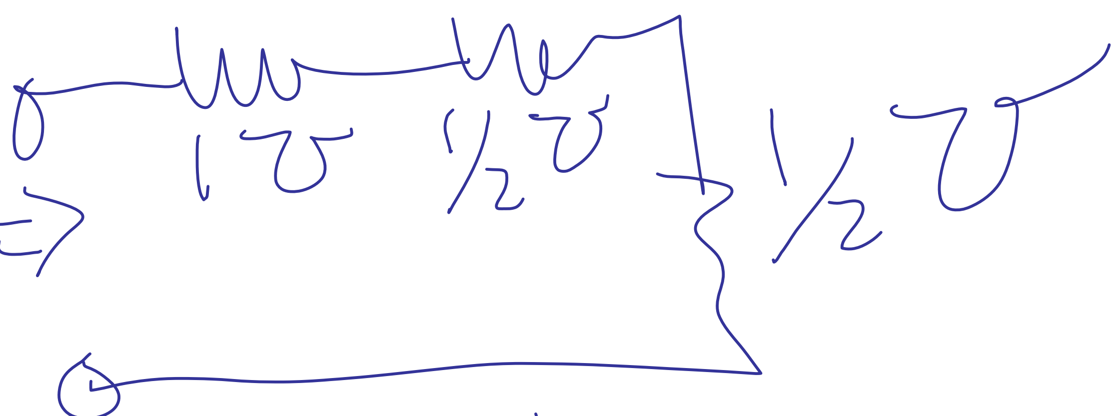
a) 1/5
b) 1/2
c) 2
d) 5
e) 10




$$R_{eq} = 1 + 2 + 2 = 5\,\Omega$$

$$\therefore G_{eq} = \frac{1}{R_{eq}} = 1/5\ \mho$$

$G_{eq} \Rightarrow$

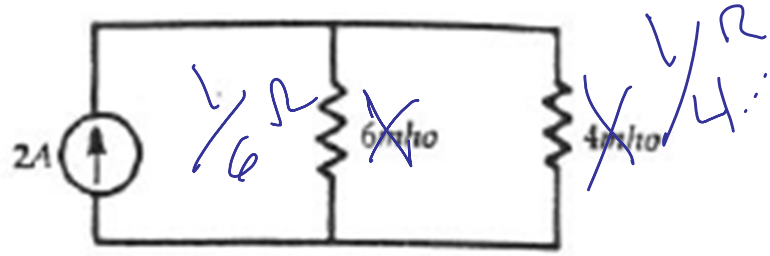



$$G_{eq} = \frac{1}{\frac{1}{1} + 2 + 2} = 1/5\ \mho$$

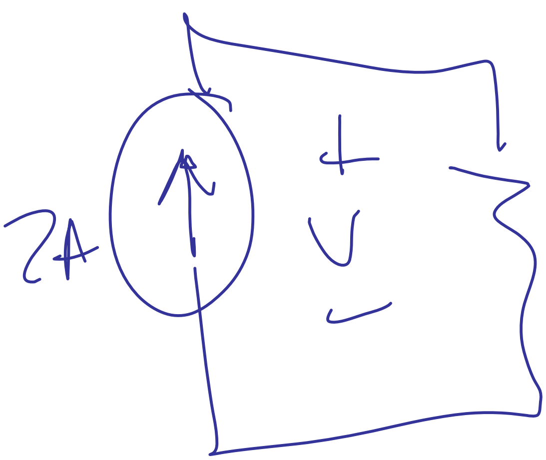
## *Practice Problems (cont.)*

6.6 The power, in watts, absorbed by the 6 mho conductance in the circuit below is

a) –.24
b) .2
c) .24
d) .48
e) 0.54




$$I_6 = \frac{1/4}{1/4 + 1/6} \cdot 2A = 1.2A$$




$$R_{eq} = \frac{1}{6+4} = \frac{1}{10}\,\Omega$$

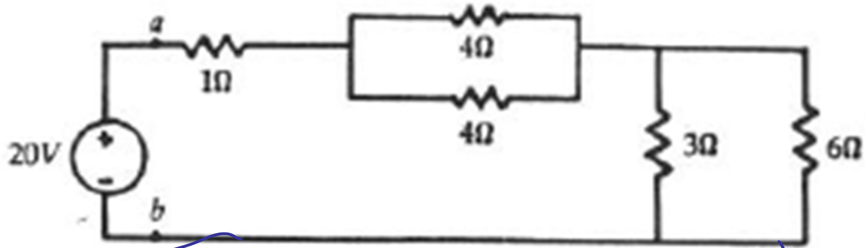
$$v = I \cdot R_{eq} = 2 \times \frac{1}{10} = 0.2V$$

$$\boxed{\text{Power absorbed} = (0.2)(1.2) = 0.24\,W}$$

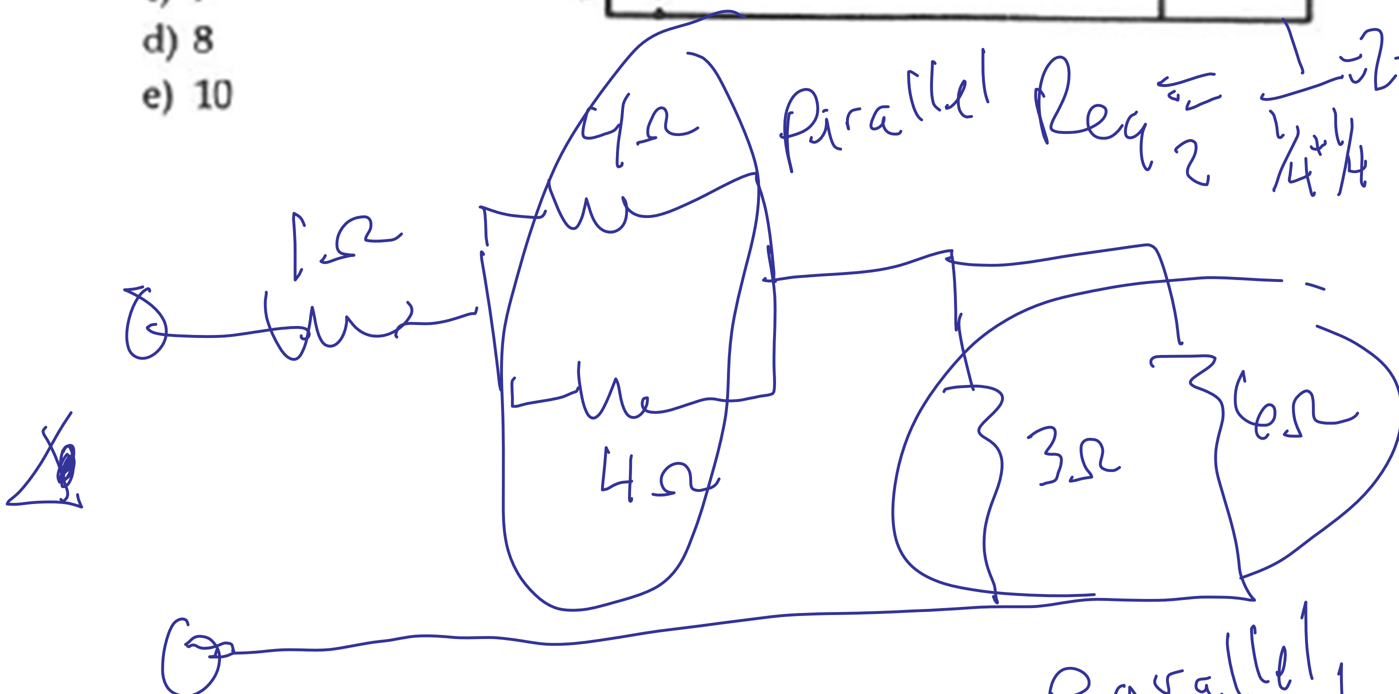
# Practice Problems (cont.)

*6.7 The equivalent resistance, in ohms, between points *a* and *b* in the circuit below is

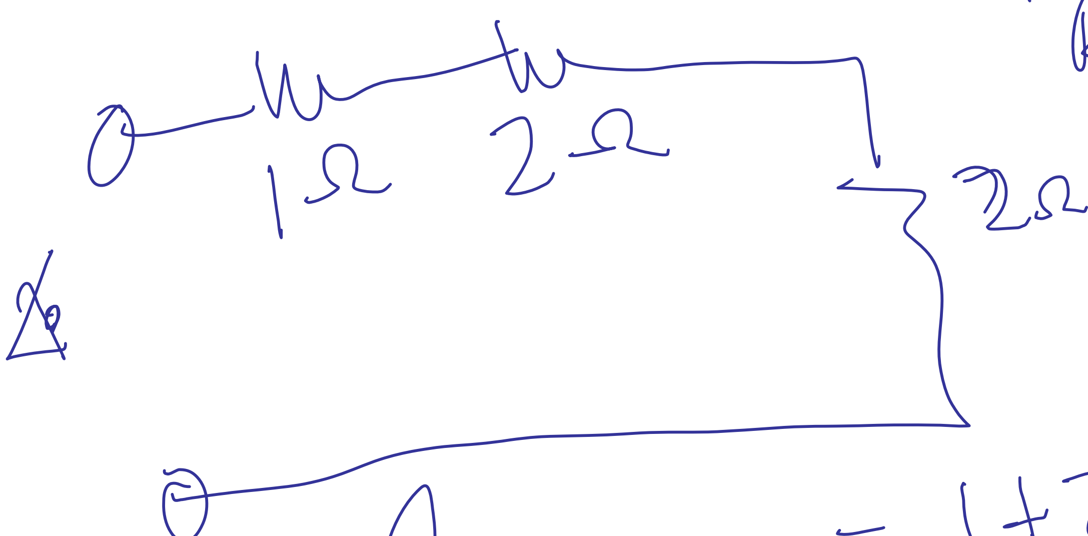
a) 3
b) 5
c) 7
d) 8
e) 10




parallel $Req_2 = \frac{1}{\frac{1}{4}+\frac{1}{4}} = 2\Omega$




parallel $Req_1 = \frac{1}{\frac{1}{3}+\frac{1}{6}}$

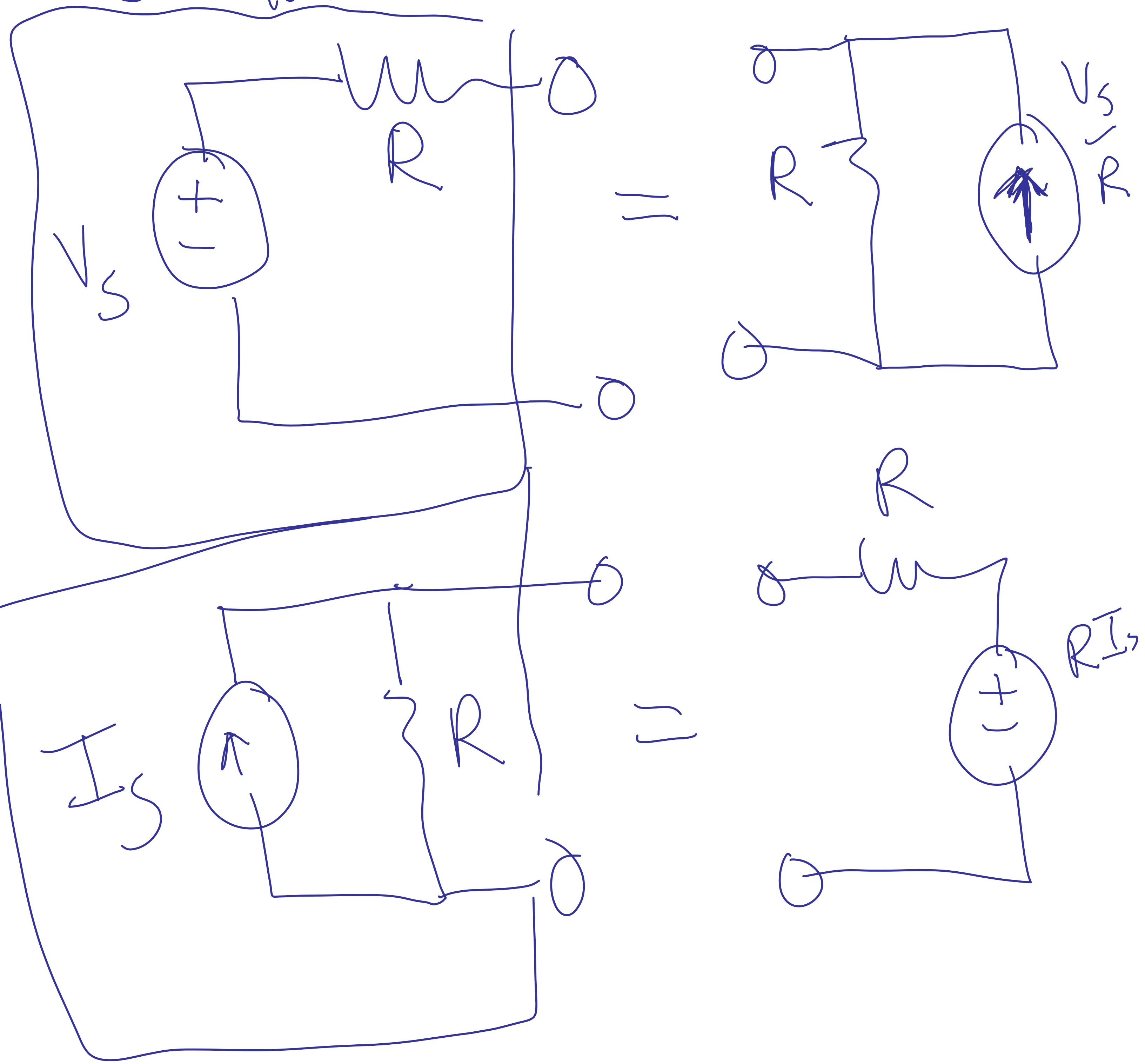



$= 2\Omega$

$Req_{whole} = 1+2+2 = 5\Omega$

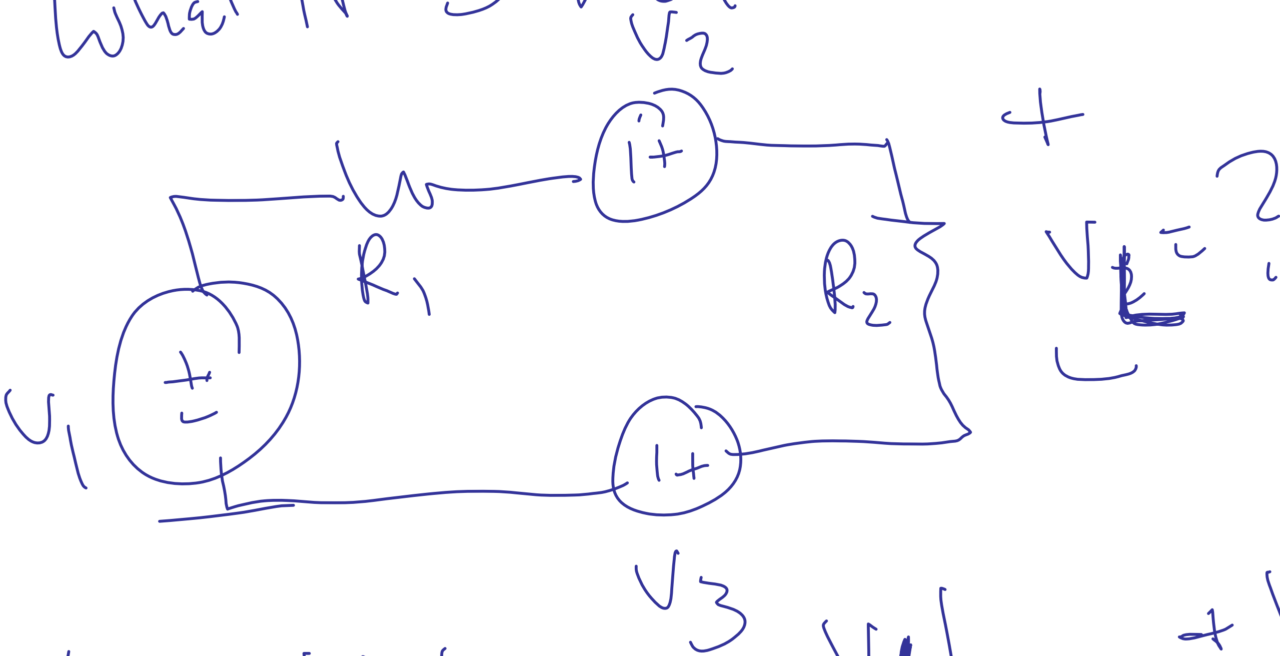
# FE Trick – Source Transformation

Common FE situation



# *FE Trick - Superposition*

What if I had




$$V_L = V_L\Big|_{\text{due to } V_1} + V_L\Big|_{\text{due to } V_2} + V_L\Big|_{\text{due to } V_3}$$

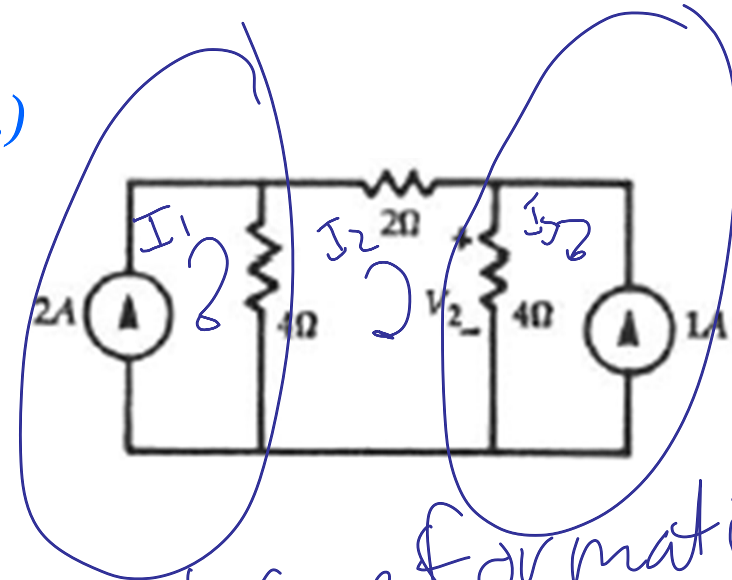
Kill a voltage source → replace by short circuit

Kill a current source → replace by open circuit

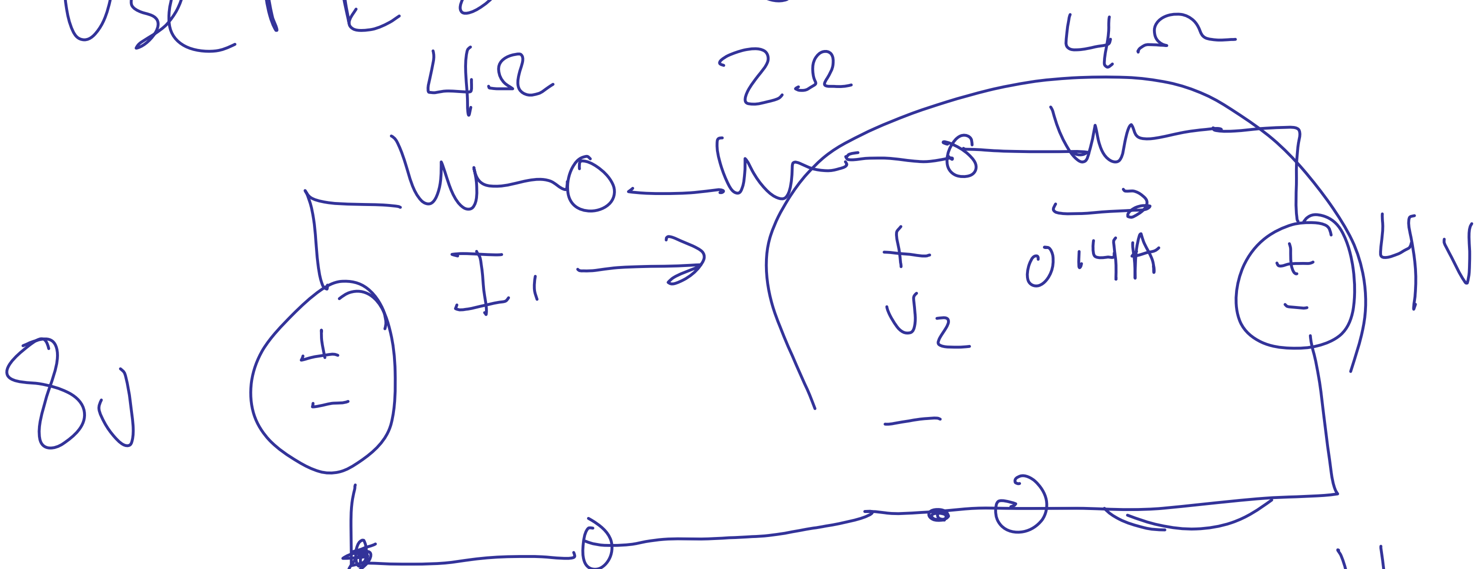
## *Practice Problems (cont.)*

6.8 The voltage $V_2$ is

a) 6.4
b) 4.0
c) 2.0
d) 5.6
e) 3.0




Use FE source transformation




KVL $-8 + 4I_1 + 2I_1 + 4I_1 + 4 = 0$

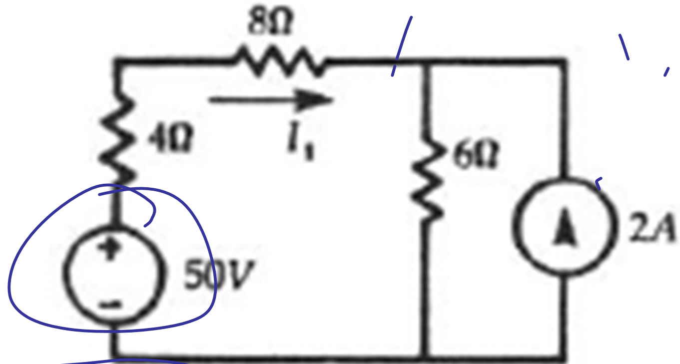
$I_1 = 0.4A$

KVL for $V_2$: $-V_2 + 4(0.4A) + 4 = 0$

$V_2 = 5.6V$

## Practice Problems (cont.)

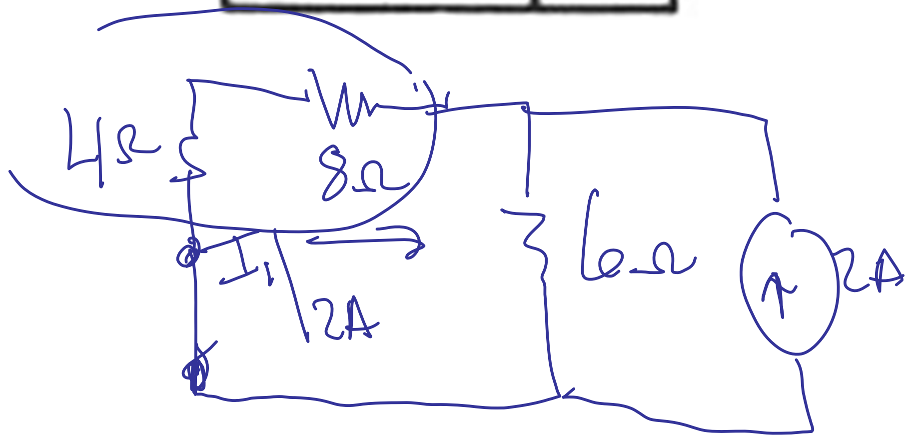
6.9 Find $I_1$ in amperes.




Kill

a) 4.0
b) 2.0
c) 4.11
d) 2.11
e) 3.0

$R_{eq} = 12\Omega$




Superposition:

$$I_1 = I_1\Big|_{\text{due to } 50V} + I_1\Big|_{\text{due to } 2A} = -\frac{6}{12+6}\,2A = -0.677$$

$$I_1\Big|_{50V} = \frac{50V}{4+6+8} = 2.777$$

$$2.777 - 0.6777 = 2.11A$$

*Take a Break*

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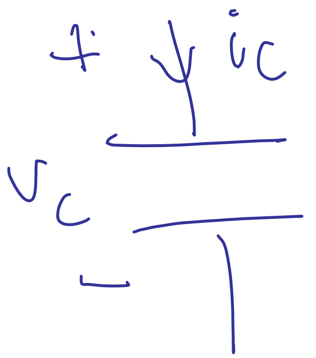
# *AC Circuits*

Complex Numbers: i or j?

$\sqrt{-1} = j$

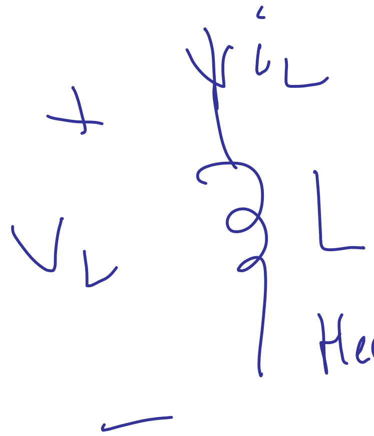
Components: New components:

Capacitor




$i_c = C \frac{dv_c}{dt}$

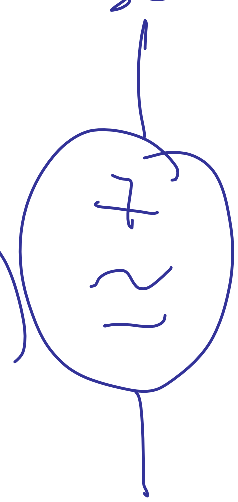
Inductor




$v_L = L \frac{di_L}{dt}$

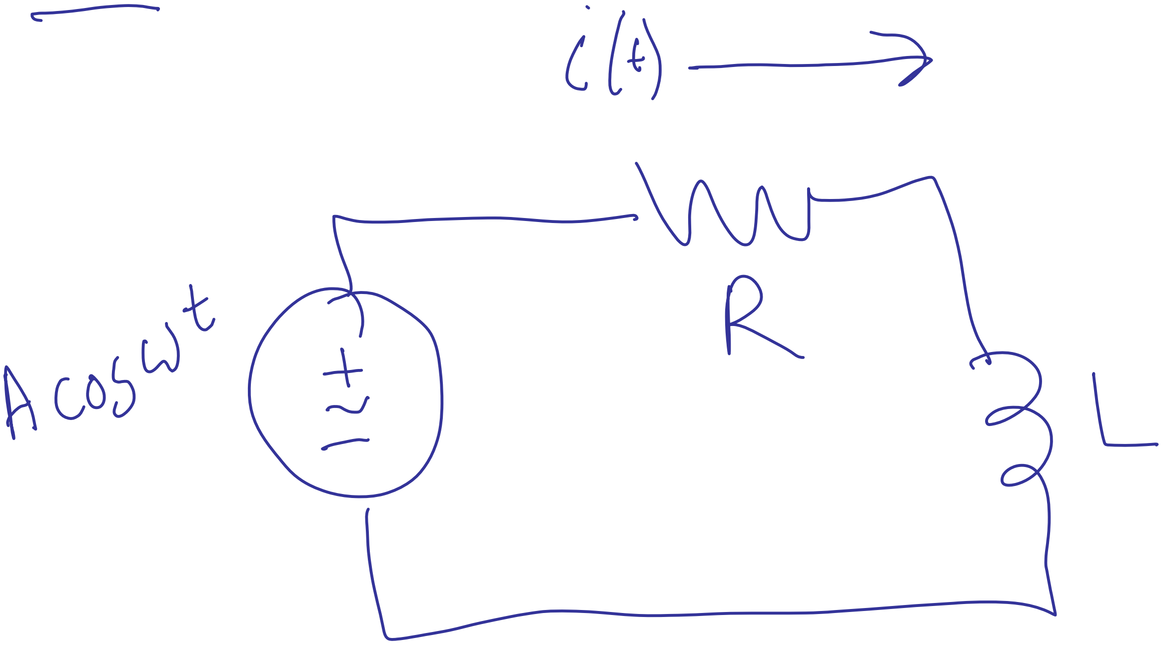
Excitation A.C. voltage or current is sinusoidal

$A\cos(\omega t + \theta)$ volts



Basic Idea: we are interested in the steady-state sol'n of the circuit (i.e., the sol'n that exists after the transient or decaying exponential sol'n has died out). We could always do the same type analysis and obtain systems of differential equations (MA 214) and solve them. But there is an easier way using Phasors (a shortcut that enables us to use DC ckt analysis tools with complex sources + elements

Ex




Do a KLV:

$$-A\cos\omega t + R\,i(t) + L\frac{di}{dt} = 0$$

This is a first order linear ordinary differential equation. It has a forced and natural sol'n (particular + homogeneous). We are interested in the <u>Forced</u> sol'n. Here is an easier way to find this

*New Terms*

Z – Impedance which is complex Ω

R – Real part of Z = Resistance Resistance Ω

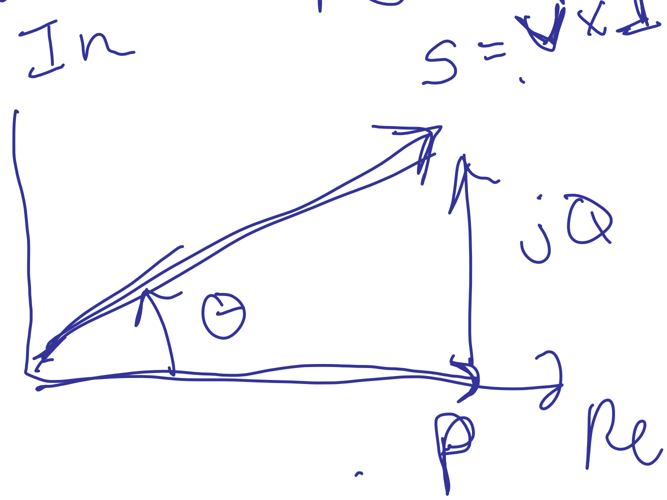
X – Imaginary Part = Reactance Ω

Y – Admittance = $\frac{1}{Z}$ units are ℧

S – Complex power = Phasor of V time Phasor of I = P + jQ

Q – Reactive power = Imaginary part of complex power

P – Real power = Real part of S



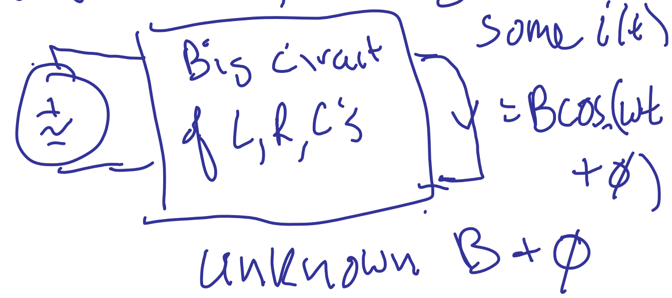
# Key to Analyzing AC Circuits: Phasors

**Basic Idea:** Apply a sinusoidal source to a circuit of R's, L's, and C's. In steady-state, all voltages and currents will be sinusoids of the frequency but with different (unknown) magnitudes and phases.

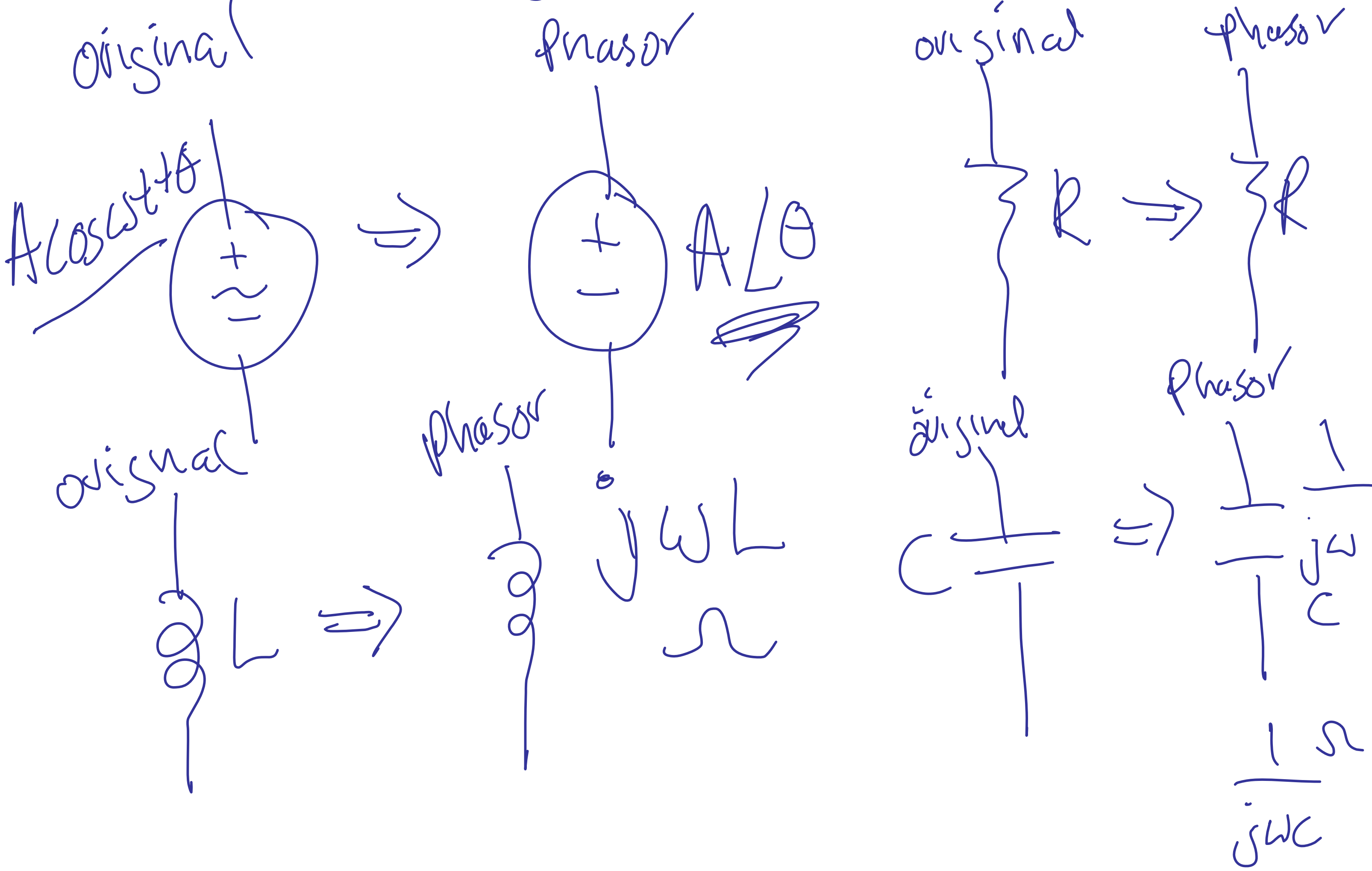
B

**How to find magnitude and phase?**

φ




Replace ckt by phasor equivalent

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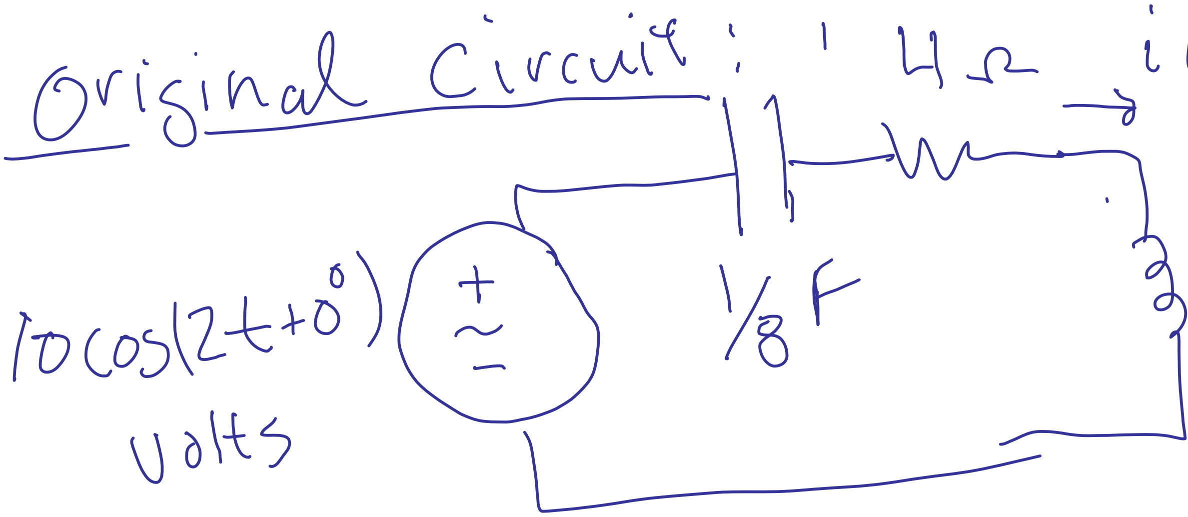
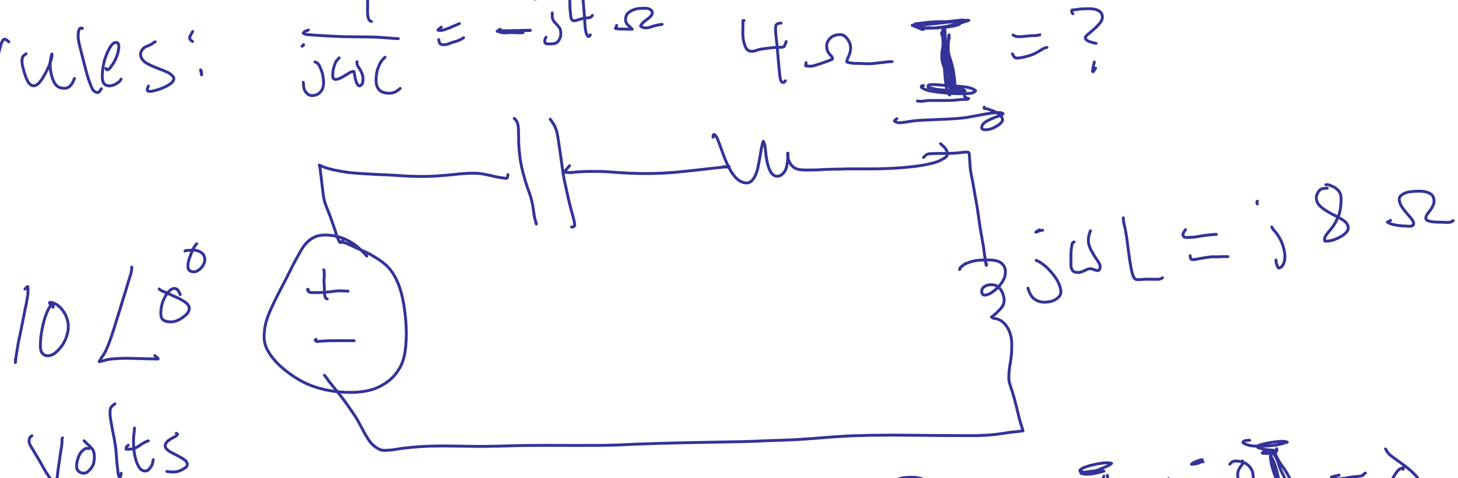
Let's do an easy example first:

Original circuit: $i(t) = ?$ (in steady-state)




Convert to a phasor ckt using our rules: $\frac{1}{j\omega C} = -j4\,\Omega$




$\therefore$ KVL: $-10\angle 0^\circ + (-j4)\bar{I} + 4\bar{I} + j8\bar{I} = 0$

$$\bar{I} = \frac{10\angle 0^\circ}{(4 + j4)} = \frac{10\angle 0^\circ}{4\sqrt{2}\angle 45^\circ} = \frac{5\sqrt{2}}{4}\angle -45^\circ$$

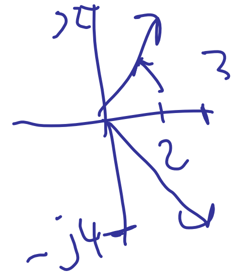
$$\Rightarrow i(t) = \frac{5\sqrt{2}}{4}\cos(2t - 45^\circ) \text{ Amps}$$

# Practice Problems (cont.)

## AC CIRCUITS—SINGLE PHASE

*6.10 (2 + $j$2) (3 − $j$4) is most nearly

a) 6.0 $\angle -21.8°$
b) ~~14.1 $\angle -21.8°$~~
c) 14.1 $\angle -8.1°$
d) 28.0 $\angle -8.1°$
e) 46.0 $\angle -8.1°$




$5\angle -53.1°$

$(2\sqrt{2}\angle 45°)(5\angle -53.1°)$

$10\sqrt{2}\angle -8.1°$

$14.1\angle -8.1°$

*Average Value and Root Mean Square (RMS)*

Average Value $= \frac{1}{T}\int_0^T x(t)\,dt$

Root Mean Square Value

or RMS value $= \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$

If $x(t) = A\cos\omega t$ (sinusoid)

Then RMS value $= \sqrt{\frac{1}{\frac{2\pi}{\omega}}\int_0^{2\pi/\omega} A^2\cos^2\omega t\,dt}$

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Half angle identity:

$$\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow \cos^2\theta = \tfrac{1}{2}\cos 2\theta + \tfrac{1}{2}$$

$$\therefore \text{RMS value} = \sqrt{\frac{2\pi A^2}{\omega}\int_0^{\omega/2\pi}\left(\tfrac{1}{2}\cos 2\omega t + \tfrac{1}{2}\right)dt}$$

Note the average of $\frac{1}{2}\cos 2\omega t$ over two complete periods is zero !!

$$\therefore \text{RMS value} = \sqrt{\frac{2\pi A^2}{\omega}\int_0^{\omega/2\pi}\tfrac{1}{2}\,dt} =$$

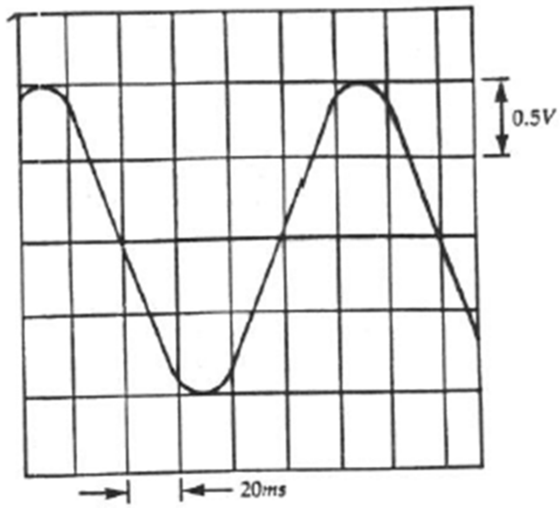
$$\sqrt{\frac{2\pi}{\omega}A^2\left(\frac{\omega}{2\pi}\right)\cdot\frac{1}{2}} = \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}}$$

$\therefore$ The RMS value of any sinusoid is the amplitude divided by $\sqrt{2}$

# *Practice Problems (cont.)*

*6.11 The following sinuisoid is displayed on an oscilloscope. The RMS voltage and the radian frequency are most nearly

a) 1, 8.33
b) .7071, 52.36
c) 1.4142, 52.36
d) 2, 8.33
e) 2, 52.36




The Amplitude A = 2 divisions

$= 2 \times 0.5\, V/div = 1$ volt

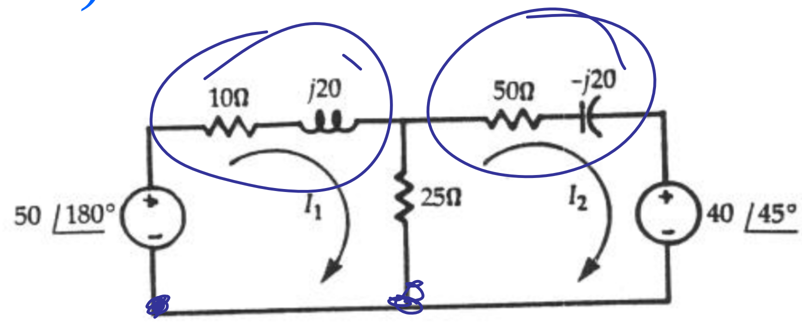
$\therefore$ RMS Voltage $= \frac{1}{\sqrt{2}} = 0.707$

The period is 6 divisions $\times \frac{20\,ms}{division}$

$= 120\,ms \Rightarrow \omega = \frac{2\pi}{period} = \frac{2\pi}{120\,ms} = 52.36$ rad/sec

# *Practice Problems (cont.)*

6.12 Find $I_2$ in amperes.




a) 0.29 + j0.68
b) −0.12 + j0.69
c) −0.82 − j0.37
d) 1 − j2
e) −3.33 − j4.50

Mesh Analysis

KVL's

L1: $-50\angle 180^\circ + 10I_1 + j20I_1 + 25(I_1 - I_2) = 0$

L2: $25(I_2 - I_1) + 50I_2 - j20I_2 + 40\angle 45^\circ = 0$

$$\boxed{\begin{bmatrix} 35+j20 & -25 \\ -25 & 75-j20 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 50\angle 180^\circ \\ 40\angle 225^\circ \end{bmatrix}}$$

$\therefore\ I_1 = -1.63 + j0.667$

$I_2 = -0.822 - j0.374$

a = 35.0000 +20.0000i -25.0000
-25.0000 75.0000 -20.0000i

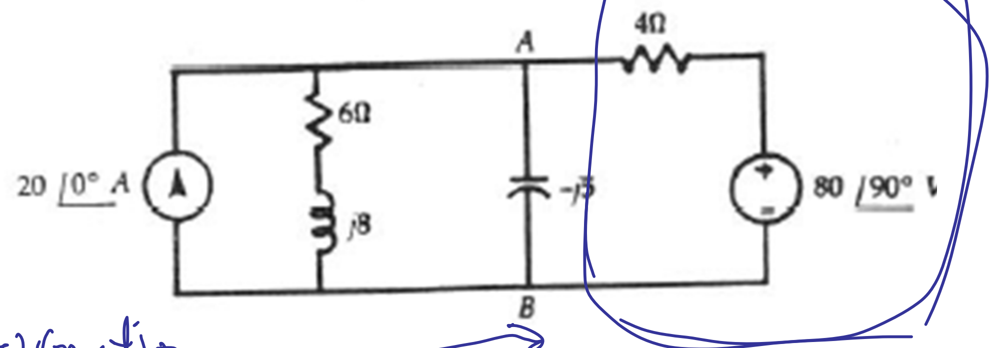
-50.0000
-28.2843 -28.2843i

I = -1.6348 + 0.6670i
-0.8223 - 0.3741i

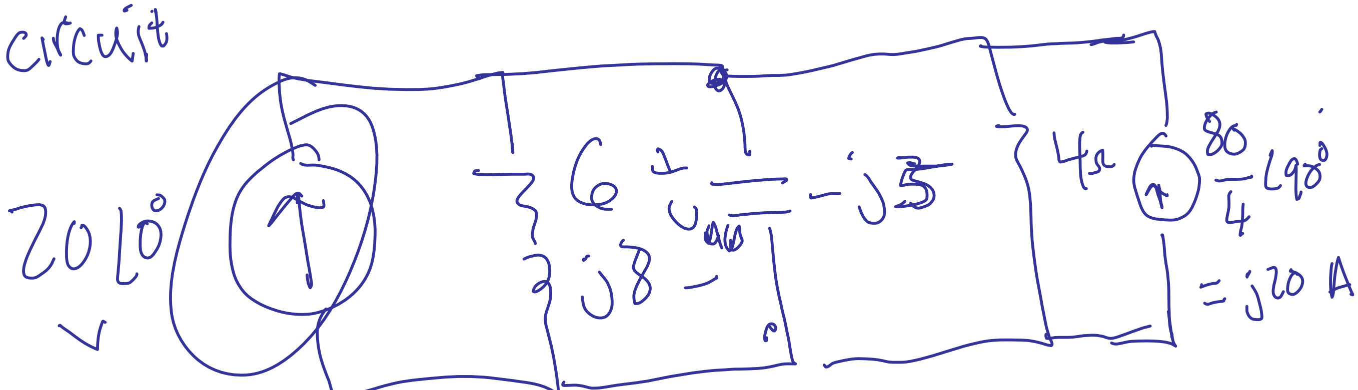
# Practice Problems (cont.)

*6.13 Calculate the magnitude of the node voltage $V_{AB}$.

a) 85.1
b) 77.2
c) 68.8
d) 92.2
e) 102.2

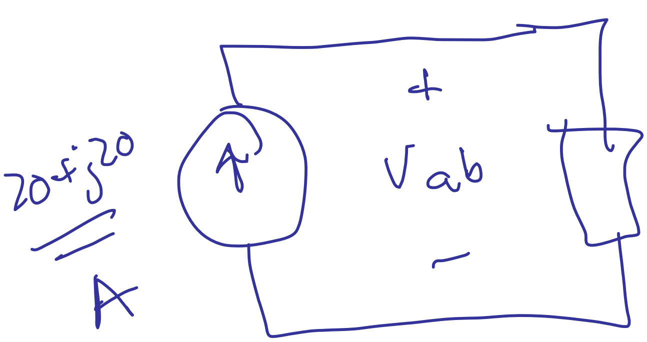



Use a source transformation → parallel circuit




Note current sources in parallel add
voltage sources in series add

Equivalent circuit:




$$Z_{eq} = \frac{1}{\frac{1}{6+j8} + \frac{1}{-j5} + \frac{1}{4}} = 2.81 - j1.09\ \Omega$$

$$V_{AB} = I \times Z_{eq} = (20+j20)(2.81-j1.09) = 77.1 + j34.4$$

zeq = 1/(1/(6+j*8)+1/(-j*5)+1/4)
zeq = ~~1.9341 - 1.5806i~~
2.8 - j1.09

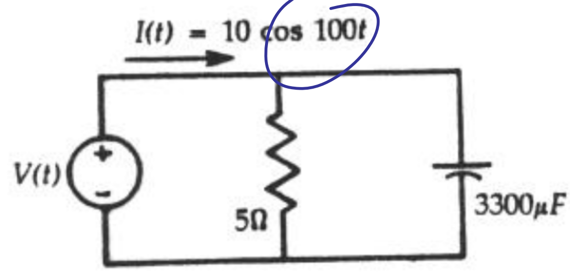
v=(20+j*20)*zeq
~~v = 70.2946 + 7.0711i~~
77.1+j34.4

Vabs =70.6494

$|V_{AB}| = 85.1$

# *Practice Problems (cont.)*

6.14 The peak value of $V(t)$ in the circuit shown is approximately




a) 2.0
b) 3.68
c) 25.9
d) 50.0
e) 71.6

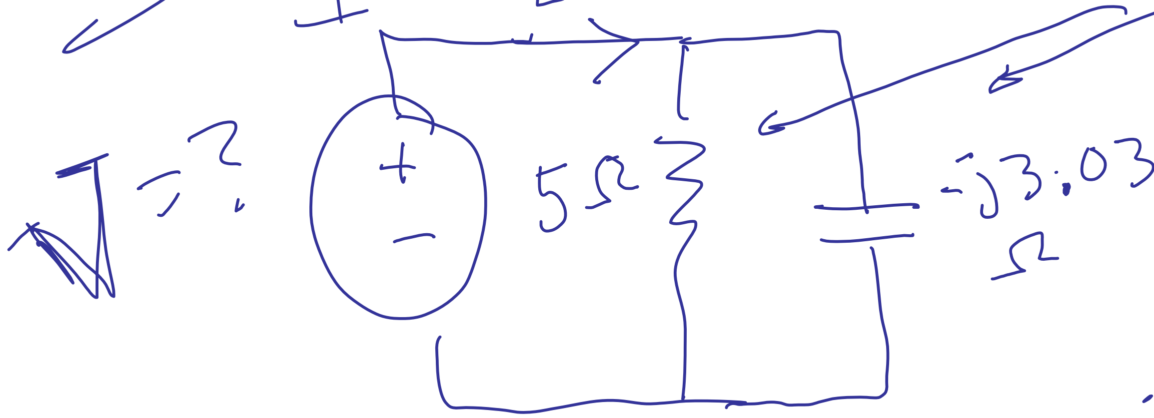
$100 \times 3300 \times 10^{-6}$

$\dfrac{1}{j\,33 \times 10^{-2}}$

Phasors

combine in parallel

$I = 10\angle 0^\circ$




$$Z_{eq} = \frac{1}{1/5 + \frac{1}{-j3.03}} = 1.3 - j2.2\ \Omega$$

$$\therefore V = I \times Z_{eq} = 10\angle 0^\circ (1.3 - j2.2)$$

$$= 13 - j22 = 25.9\angle -58.7^\circ$$

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*AC Power*

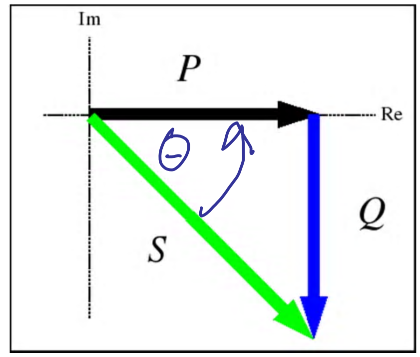
**Instantaneous Power =**

**Complex power** $S$ = P + jQ = **V x I**

**Real power** ($P$) [Unit: W]

**Reactive power** ($Q$) [Unit: var]

**Apparent Power** ($|S|$) [Unit: VA]



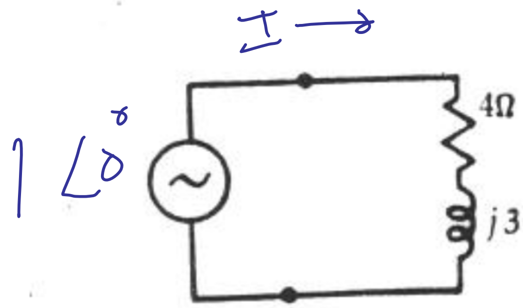

PF = **Power Factor** – your house (and an Industry) have lots of appliances and motors that present an inductive load to the power company. The more inductance you have, the more reactive (wasted) power you have. Ideally, the power company wants <u>NO</u> reactive power delivered

The power Factor is a measure of how close to a pure resistance a load is: $PF = \cos\theta = \frac{P}{|S|}$

# *Practice Problems (cont.)*

*6.15 The power factor of the circuit shown is most nearly

a) 0.5
b) 0.6
c) ~~0.7~~
d) 0.8
e) 0.9




$$Z_{eq} = 4 + j3 = 5\angle 56.1^\circ \Rightarrow \theta = 56.1^\circ$$

$$\therefore PF = \cos\theta = \cos(56.1^\circ) = 4/5$$

Another way to solve is input a voltage of $1\angle 0^\circ$ volts and measure $I$

$$\therefore I = \frac{1\angle 0^\circ}{4+j3} = \frac{1}{5}\angle -56.1^\circ$$

$$\therefore S = \bar{V} \times \bar{I} = 1\angle 0^\circ \times \frac{1}{5}\angle -56.1^\circ = \frac{1}{5}\angle -56.1^\circ$$

$$\therefore PF = \cos\theta = \cos(-56.1^\circ) = 0.8$$

Note: $S = \frac{1}{5}\angle -56.1^\circ = \underset{P}{0.8} - j\,\underset{Q}{0.6}$

>> cos(angle(4+j*3))

ans =0.8000

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Thank You and Good Luck!