Dynamics FE Review

1-D Kinematics

$$\boxed{v = \frac{ds}{dt}, \quad a = \frac{dv}{dt}, \quad a\,ds = v\,dv}$$

<Example> $s = 2t^3 + 4t \quad v =$ ______ , $a =$ ______

<Example> $a = 3t^2 + 1 \quad$ I.C's $v(0) = 2$, $s(0) = 3$

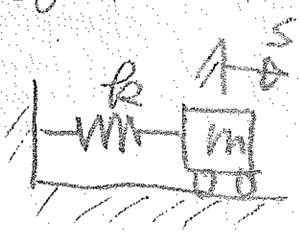
Find $v(t)$, $s(t)$

Sol:

<Example> $v = 2s \quad a =$ ______

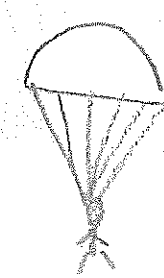
<Example> $a = -\frac{1}{2}s \quad$ I.C. $v_0 = 2$ at $s = 0$

Find v at $s = 1$



Sol:

<Example> $a = 9.81 - 2v \quad v_0 = 10$ m/s



A101

(i) Find v at $t = 10$ s

(ii) Find v at $s = 10$ m

2-D Kinematics

X-y Projectile Motion

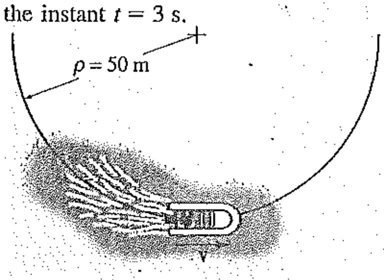
Find s (Ans: s = 1059 m)

$$\begin{cases} v_{0x} = \\ v_{0y} = \end{cases}$$

$$x = v_{0x} t$$

$$y = v_{0y} t - \frac{1}{2} g t^2$$

<example>. Starting from rest, the motorboat travels around the circular path, $\rho = 50$ m, at a speed $v = (0.2t^2)$ m/s, where t is in seconds. Determine the magnitudes of the boat's velocity and acceleration at the instant $t = 3$ s.

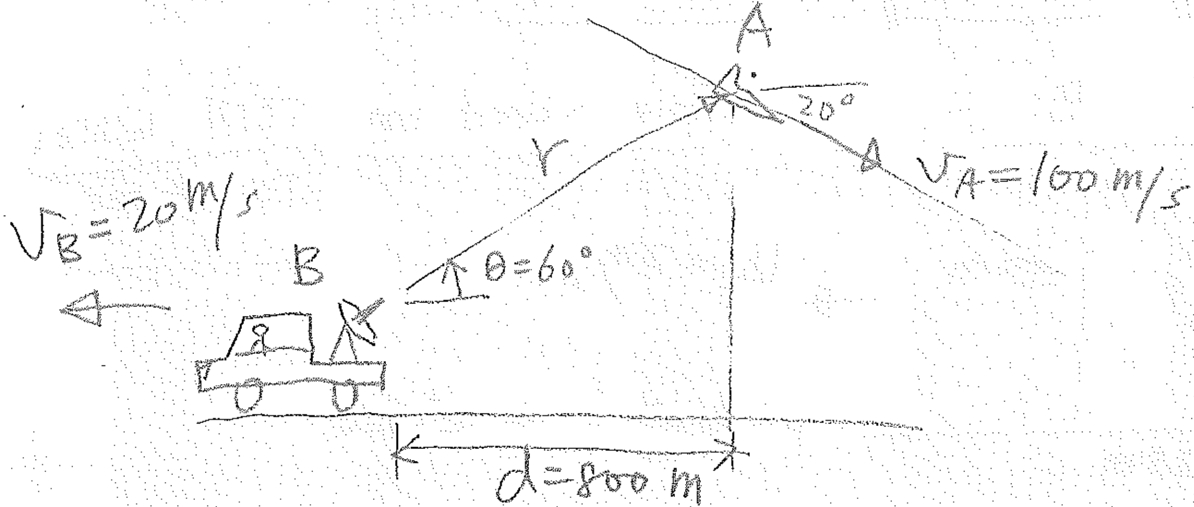


Sol: $v = 0.2t^2 \quad t = 3s \quad v = 1.8$ m/s

$$\begin{cases} a_t = \dot{v} = 0.4t \\ a_n = \dfrac{v^2}{\rho} = \dfrac{(0.2t^2)^2}{50} \end{cases} \quad t = 3s \Rightarrow \quad \begin{aligned} a_t &= 1.2 \text{ m/s}^2 \\ a_n &= \frac{1.8^2}{50} = 0.0648 \text{ m/s}^2 \end{aligned}$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{1.2^2 + 0.0648^2} = 1.2017 \text{ m/s}^2$$

r-θ Find $\dot{r}$, $\dot{\theta}$ ($\dot{r} = 27.36$ m/s $\dot{\theta} = -0.0724$ rad/s)



Sol: Find $\underset{\sim}{v}_{A/B}$ first

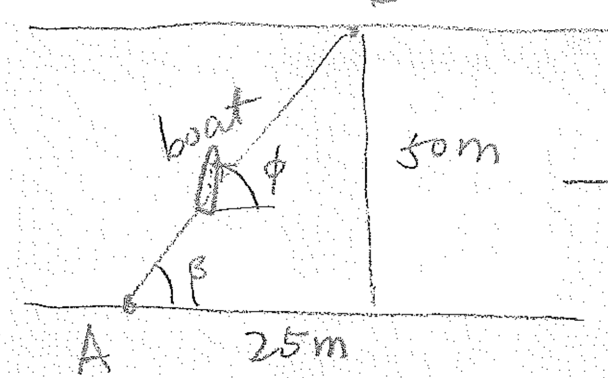
$$\underset{\sim}{v}_{A/B} = \underset{\sim}{v}_A - \underset{\sim}{v}_B$$

? =

$\underset{\sim}{v}_{A/B} =$

Decompose $\underset{\sim}{v}_{A/B}$ into polar

Relative — Crossing the river



Your speed on still water is 5 m/s

$\rightarrow v_W = 2\ m/s$

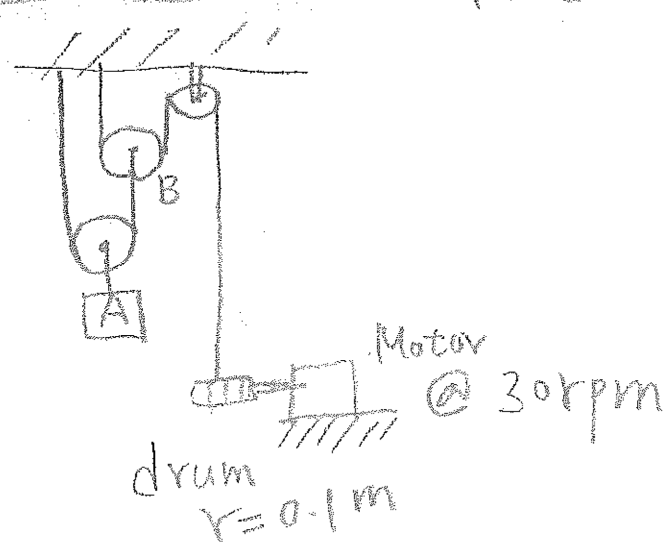
$$\beta = \tan^{-1}\frac{2}{1} = 63.435°$$

Find ϕ, v_b (Ans: $\phi = 84.4°$, $v_b = 5.56\ m/s$)

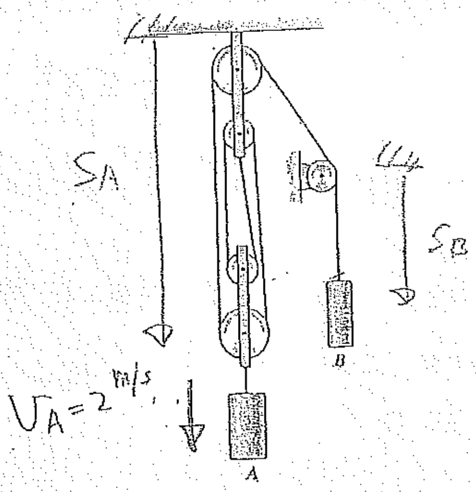
$$\underset{\sim}{v_b} = \underset{\sim}{v_w} + \underset{\sim}{v_{b/w}}$$

Constrained Motion

Find v_A (Ans: 0.078 m/s ↑)



If v_A=2 m/s downward at the instant, find the magnitude and direction of v_B.



$L = 4S_A + S_B$

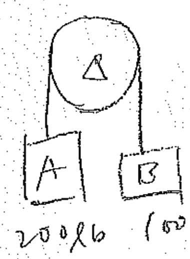
$\dot{L} = 4\dot{S}_A + \dot{S}_B = 0$

$\dot{S}_A = 2$ m/s $\quad \boxed{\dot{S}_B = -8 \text{ m/s}}$

$\boxed{V_B = 8 \text{ m/s} \uparrow}$

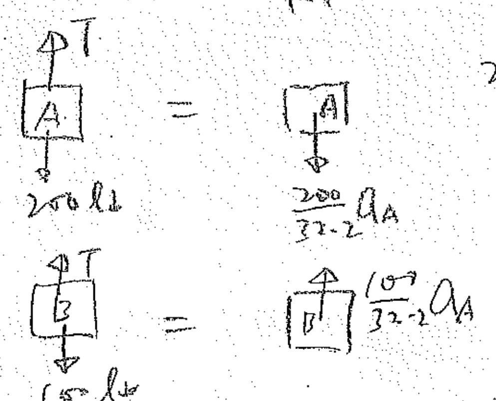
Kinetics of Particles $\quad \underset{\sim}{\Sigma F} = m\underset{\sim}{a}$

X-y



Find T : $\quad a_B = a_A$

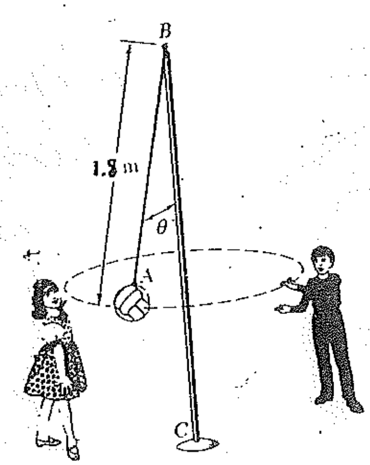
FBD $\quad$ KD



$200 - T = \frac{200}{32.2} a_A$

$T - 100 = \frac{100}{32.2} a_A$

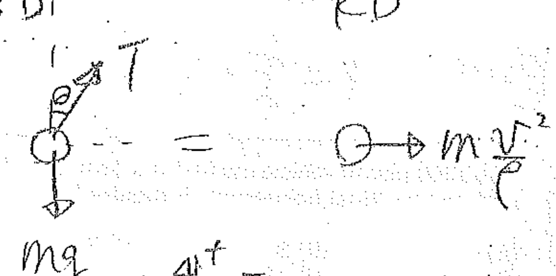
n-t



Given $\theta = 50°$, $m_A = 0.45$ kg

Find V_A and T.

Sol: FBD $\quad$ KD



$+\uparrow \Sigma F_b = 0 \quad T\cos\theta = mg$

$\rightarrow \Sigma F_n = ma_n \quad T\sin\theta = m\frac{v^2}{\rho}$

$\Rightarrow v = 4.02$ m/s

$\rho = 1.8\sin\theta$
$= 1.379$ m

Work and Energy

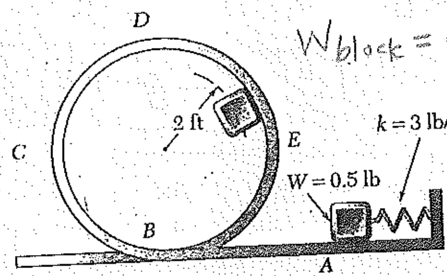
$$T_1 + V_1 + U^{\text{non}}_{1-2} = T_2 + V_2$$

$$T = \frac{1}{2} m v^2$$

$$V_e = \frac{1}{2} k x^2$$

$$V_g = mgy$$

<examples>



$W_{block} = 0.5$ lb

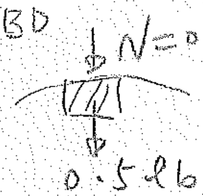
$k = 3$ lb/in $= 36$ lb/ft

no friction

Find x_{min} so that the block can finish the loop ABCD

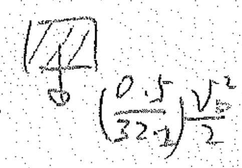
Sol: At D

FBD



=

KD



$\downarrow \Sigma F_n = m a_n$

$$0.5 = \frac{0.5}{32.2}\frac{V_D^2}{2} \qquad V_D = 8.025 \text{ ft/s}$$

From A to D

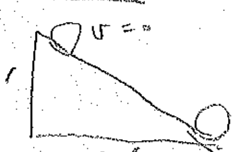
$$T_1 + V_1 + \cancel{U^{\text{non}}_{1-2}} = T_2 + V_2$$

$$\frac{1}{2}(36)x^2 = \frac{1}{2}\frac{0.5}{32.2}(8.025)^2 + 0.5(4)$$

$$x = 0.3727 \text{ ft}$$

. A 5 pound sphere moves down a frictionless plane with a vertical drop of 1 for every horizontal distance of 4. What is the velocity after the sphere has experienced a vertical displacement of 1 foot?

(A) 64.4 fps (B) 8.0 fps (C) 32.2 fps
(D) 33.1 fps (E) 1094.0 fps



$$\cancel{T_1} + V_1 + \cancel{U^{\text{non}}_{1-2}} = T_2 + V_2$$

$$5(1) = \frac{1}{2}\frac{5}{32.2}v^2$$

$$v = 8.02 \text{ ft/s}$$

Power

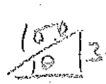
$$P = \underset{\sim}{F} \cdot \underset{\sim}{v}$$

. A 2200 pound vehicle maintains a constant 45 mph up a 3% incline. What horsepower is required?

(A) 5 (B) 8 (C) 24
(D) 72 (E) 114

$v = 45$ mph

$$F = W \sin\theta = 2200\left(\frac{3}{100}\right)$$



$$v = 45 \text{ mph} = 45(5280)/3600 = 66 \text{ ft/s}$$

$$P = Fv = 2200\left(\frac{3}{100}\right)(66) = 4356 \text{ ft}\cdot\text{lb/s} = \frac{4356}{550} \text{ hp} = 7.92 \text{ hp}$$

Work and Energy Involving Rigid Bodies

$$T_1 + V_1 + U^{non}_{1-2} = T_2 + V_2$$

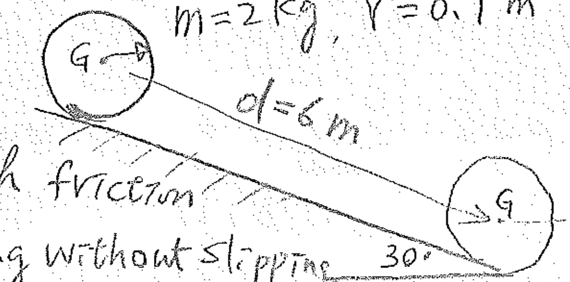
$$\boxed{T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2}$$

For fixed-axis rotation, $T = \frac{1}{2} I_O \omega^2$ $I_O = I_G + md^2$

If you can find I.C., $T = \frac{1}{2} I_{IC} \omega^2$

$$\boxed{V_g = mg y_G}$$

<example> $m = 2\,kg$, $r = 0.1\,m$ released from rest



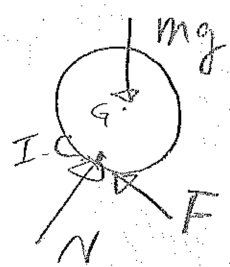
Enough friction

rolling without slipping

$v_G = ?$

$\begin{cases} \text{disk} & I_G = \frac{1}{2} m r^2 \\ \text{sphere} & I_G = \frac{2}{5} m r^2 \end{cases}$

Sol:



Usually friction does negative work.

Friction does <u>no work</u> in rolling.

Why? <u>I.C. has no velocity</u>

$$\cancel{T_1}^{\,0} + V_1 + \cancel{U^{non}_{1-2}}^{\,0} = T_2 + \cancel{V_2}^{\,0}$$

$$2(9.81)(6 \sin 30^\circ) = \frac{1}{2}(2) v_G^2 + \frac{1}{2} \underbrace{\left[\frac{1}{2}(2)(0.1)^2\right]}_{\text{if it is a disk}} \omega^2$$

Rolling $\Rightarrow$ $v_G = \omega r$

or $\omega = \frac{v_G}{r} = \frac{v_G}{0.1} = 10 v_G$

plug into the above equation $\Rightarrow$ $\boxed{v_G = 6.264\ m/s}$

Impulse and Momentum

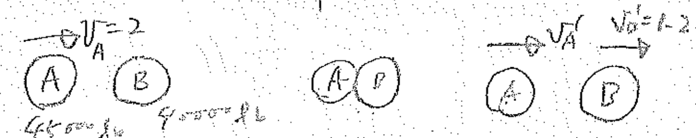
$$\underset{\sim}{L_1} + \int_{t_1}^{t_2} \Sigma \underset{\sim}{F}\, dt = \underset{\sim}{L_2} \qquad \underset{\sim}{L} = m\underset{\sim}{V} = \text{linear momentum}$$

Impact

A 45,000 pound railroad car is moving at a speed of 2 fps to the right. After colliding with a 40,000 pound car initially at rest, the 40,000 pound car moves to the right with a speed of 1.2 fps. What is the coefficient of restitution?

(A) 7.4 (B) .14 (C) 1.1
(D) .9 (E) .6

before Impact after



A+B $\underset{\sim}{L_1} = \underset{\sim}{L_2}$

$$\rightarrow + \quad \frac{45{,}000}{32.2}(2) = \frac{45{,}000}{32.2}V_A' + \frac{40{,}000}{32.2}V_B'$$

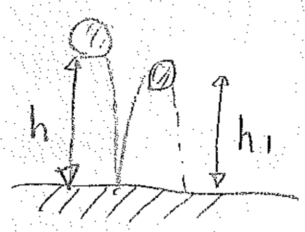
$$V_A' = 0.933 \rightarrow$$

$$e = \frac{1.2 - V_A'}{2} = \frac{1.2-0.933}{2} = 0.1335$$

Restitution

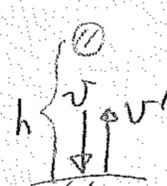
$$e = \left| \frac{\text{rel. } v \text{ of sep.}}{\text{rel. } v \text{ of app}} \right|$$

<example>



$e = ?$

Sol:



$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

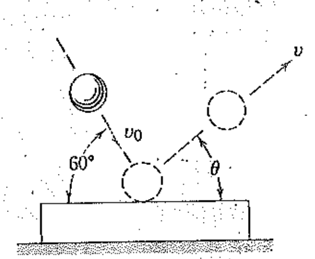
$$\frac{1}{2}mv'^2 = mgh_1$$

$$v' = \sqrt{2gh_1}$$

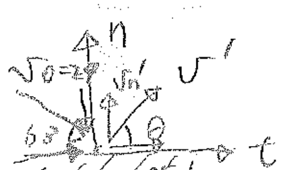
$$e = \frac{v'}{v} = \sqrt{\frac{h_1}{h}}$$

Oblique Impact

: e applies to the n dir only.



The steel ball strikes the heavy steel plate with a velocity $v_0 = 24$ m/s at an angle of 60° with the horizontal. If the coefficient of restitution is $e = 0.8$, compute the velocity v and its direction θ with which the ball rebounds from the plate.

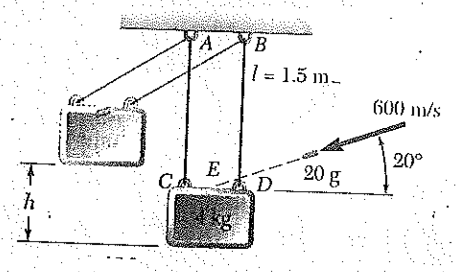


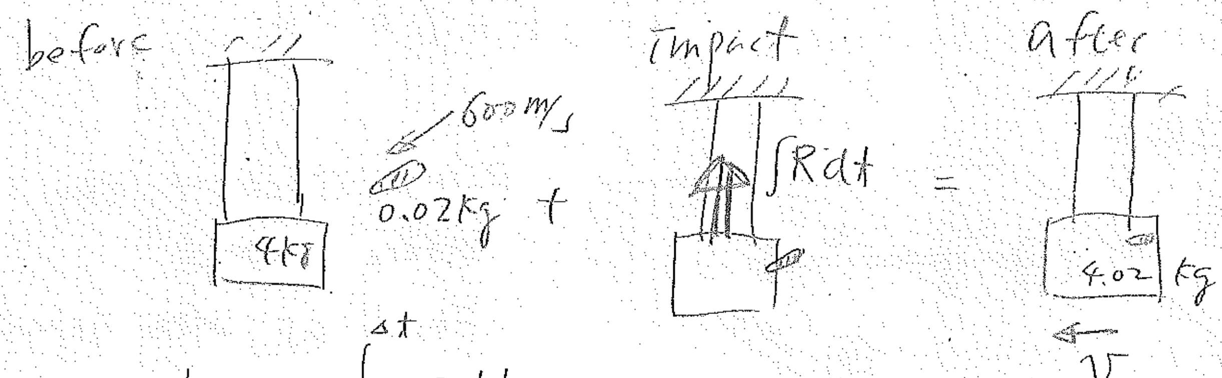
$$V_{t'} = V_t = 24\cos 60° = 12 \text{ m/s}$$

$$e = 0.8 = \frac{V_n'}{24\sin 60°} \Rightarrow V_n' = 16.63,$$

$$V = \sqrt{12^2 + 16.63^2} = 20.5 \text{ m/s} \qquad \theta = 54.2°$$

A 20-g bullet fired into a 4-kg wooden block suspended from cords AC and BD penetrates the block at point E, halfway between C and D, without hitting cord BD. Determine (a) the maximum height h to which the block and the embedded bullet will swing after impact, (b) the total impulse exerted on the block by the two cords during the impact.





$$\underset{\sim}{L_1} + \int_0^{\Delta t} \underset{\sim}{\Sigma F}\, dt = \underset{\sim}{L_2}$$

$\overset{+}{\leftarrow}$ $\quad 0.02(600)\cos 20^\circ = 4.02\, v$

$$\boxed{v = 2.805}\ m/s$$

$+\uparrow$ $\quad -0.02(600)\sin 20^\circ + \int_0^{\Delta t} R\,dt = 0$

$$\boxed{\int_0^{\Delta t} R\,dt = 4.1\ N\cdot s}$$

After impact

$$T_1 + V_1 + \cancel{U_{1-2}}^{\,\text{non}} = T_2 + V_2$$

$$\frac{1}{2}(4.02)(2.805)^2 = 4.02(9.81)\,h$$

$$\boxed{h = 0.401\ m}$$

Angular Impulse and Momentum

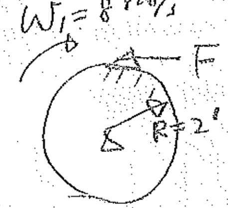
$$\underset{\sim}{H}_1 + \int_{t_1}^{t_2} \Sigma \underset{\sim}{M}\, dt = \underset{\sim}{H}_2$$

$$\underset{\sim}{H} = \underset{\sim}{r} \times m\underset{\sim}{v} = I_o \underset{\sim}{\omega}$$

for rigid bodies

. . A 1-slug wheel with a 2-foot radius rotates at 8 radians per second. What is the tangential force required to stop rotation in 5 seconds?

(A) .8 lb (B) 1.6 lb (C) 4.0 lb
(D) 8.0 lb (E) 10.0 lb



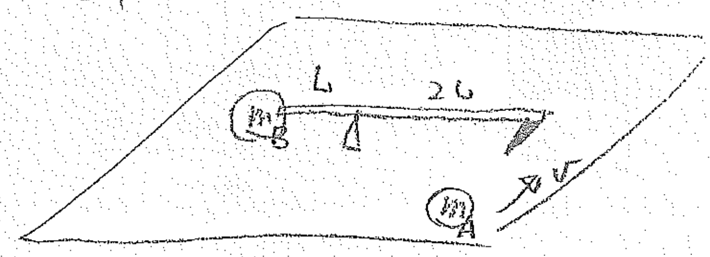
wheel

$$I_o = \frac{1}{2} m R^2 = \frac{1}{2}(1)(2)^2 = 2 \text{ slug ft}^2$$

$$2(8) - F(2)(5) = 0$$

$$F = 1.6 \text{ lb}$$

Impact



Ball A sticks to the lever after impact

Find ω after impact

$$\underset{\sim}{H}_1 = \underset{\sim}{H}_2$$

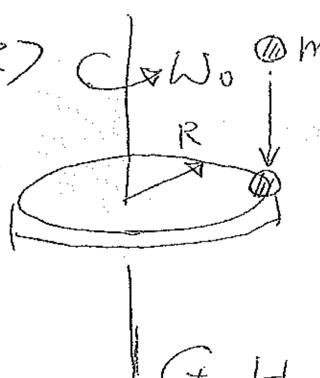
$$m_A v(2L) = m_B(L\omega)L + m_A(2L\omega)(2L)$$

$$\omega = \frac{2 m_A v}{m_B L + 4 m_A L}$$

<Example>

disk

M



m drops and sticks to disk at R

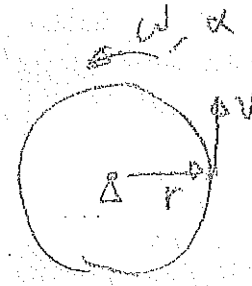
Find new ω

$$H_1 = H_2$$

$$\left(\frac{1}{2}MR^2\right)\omega_0 = \left(\frac{1}{2}MR^2 + mR^2\right)\omega$$

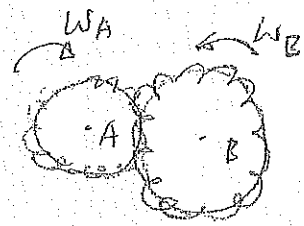
$$\omega = \left(\frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + mR^2}\right)\omega_0$$

Rotation:



$$\begin{cases} v = r\omega \\ a_t = r\alpha \\ a_n = \dfrac{v^2}{r} = r\omega^2 \end{cases}$$

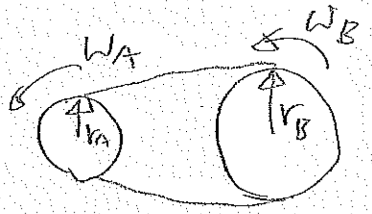
Gears



Compatibility:

$r_A \theta_A = r_B \theta_B$

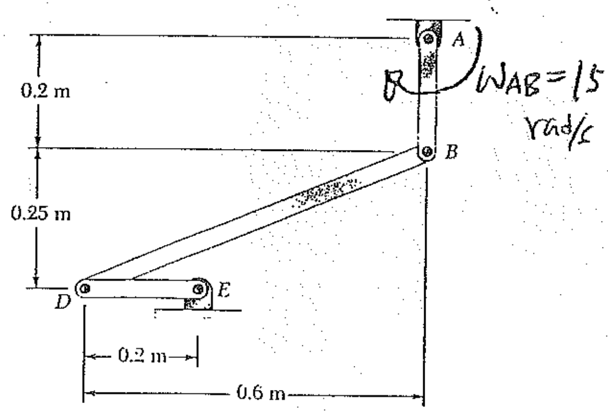
Belts



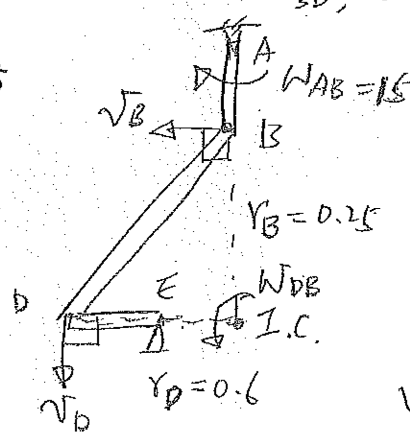
$r_A \omega_A = r_B \omega_B$

$r_A \alpha_A = r_B \alpha_B$

Instantaneous Center of zero velocity (I.C.)



Find ω_{BD}, ω_{DE}



$v_B = 15(0.2)$
$= 3$ m/s

$\omega_{DB} = \dfrac{3}{0.25}$
$= 12$ rad

$v_D = 12(0.6)$
$= 7.2$ m/s

$\omega_{DE} = \dfrac{7.2}{0.2} = 3$

Find ω_{BD} & ω_{DE}