Method of Superposition

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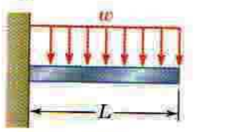
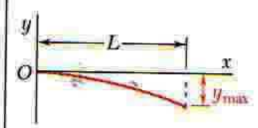
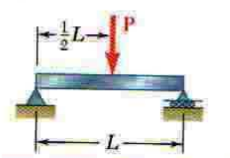
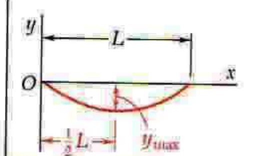
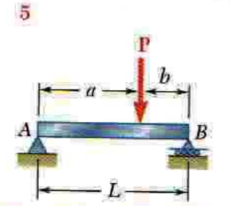
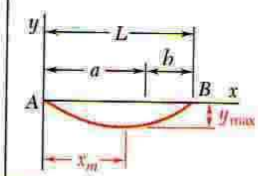
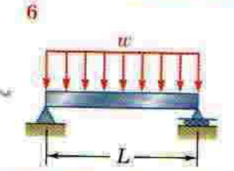
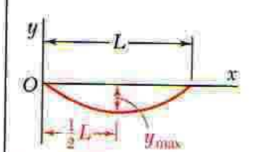
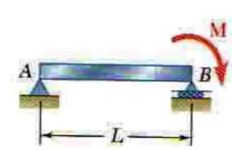
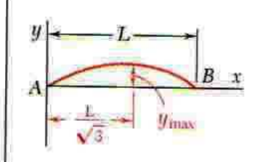
APPENDIX D Beam Deflections and Slopes

Beam and Loading	Elastic Curve	Maximum Deflection	Slope at End	Equation of Elastic Curve
1 		$-\dfrac{PL^3}{3EI}$	$-\dfrac{PL^2}{2EI}$	$y = \dfrac{P}{6EI}(x^3 - 3Lx^2)$
2 		$-\dfrac{wL^4}{8EI}$	$-\dfrac{wL^3}{6EI}$	$y = -\dfrac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
3 		$-\dfrac{ML^2}{2EI}$	$-\dfrac{ML}{EI}$	$y = -\dfrac{M}{2EI}x^2$
4 		$-\dfrac{PL^3}{48EI}$	$\pm\dfrac{PL^2}{16EI}$	For $x \leq \frac{1}{2}L$: $y = \dfrac{P}{48EI}(4x^3 - 3L^2x)$
5 		For $a > b$: $-\dfrac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}EIL}$ at $x_m = \sqrt{\dfrac{L^2-b^2}{3}}$	$\theta_A = -\dfrac{Pb(L^2-b^2)}{6EIL}$ $\theta_B = +\dfrac{Pa(L^2-a^2)}{6EIL}$	For $x < a$: $y = \dfrac{Pb}{6EIL}[x^3 - (L^2-b^2)x]$ For $x = a$: $y = -\dfrac{Pa^2b^2}{3EIL}$
6 		$-\dfrac{5wL^4}{384EI}$	$\pm\dfrac{wL^3}{24EI}$	$y = -\dfrac{w}{24EI}(x^4 - 2Lx^3 + L^3x)$
7 		$\dfrac{ML^2}{9\sqrt{3}EI}$	$\theta_A = +\dfrac{ML}{6EI}$ $\theta_B = -\dfrac{ML}{3EI}$	$y = -\dfrac{M}{6EIL}(x^3 - L^2x)$

Normal Stress

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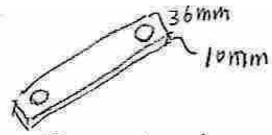
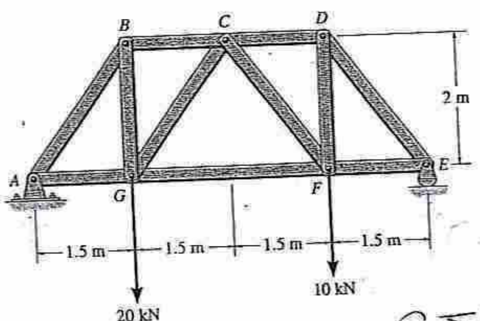


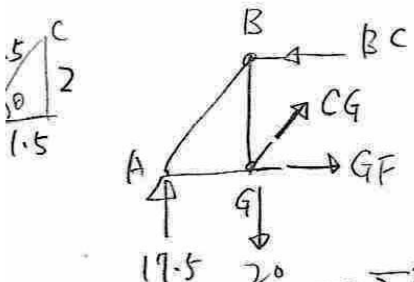
(1) Each member of the truss has a uniform rectangular cross section 10 mm thick and 36 mm wide. Each pin has a 16-mm diameter. Determine the maximum value of the average normal stress in (a) member BC, (b) member CG, and (c) member GF. (25%)



$+\circlearrowleft \sum M_E = 0$

$A_y(6) - 20(4.5) - 10(1.5) = 0$

$\boxed{A_y = 17.5\ kN}$



$+\circlearrowleft \sum M_G = 0$

$BC(2) - 17.5(1.5) = 0$ $\quad \boxed{BC = 13.125\ kN\ (C)}$

$+\uparrow \sum F_y = 0$

$17.5 - 20 + CG\left(\frac{2}{2.5}\right) = 0$ $\quad \boxed{CG = 3.125\ kN\ (T)}$

$\xrightarrow{+} \sum F_x = 0$

$GF + 3.125\left(\frac{1.5}{2.5}\right) - 13.125 = 0$ $\quad \boxed{GF = 11.25\ kN\ (T)}$

(a) $\sigma_{BC} = \frac{-13.125\ kN}{36(10)(10^{-6})\ m^2} = \boxed{-36.458\ MPa}$

(b) $\sigma_{CG} = \frac{3.125\ kN}{(36-16)(10)(10^{-6})\ m^2} = \boxed{15.625\ MPa}$

(c) $\sigma_{GF} = \frac{11.25\ kN}{(36-16)(10)(10^{-6})\ m^2} = \boxed{56.25\ MPa}$

Shearing Stress

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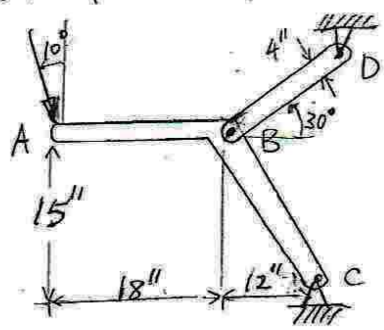
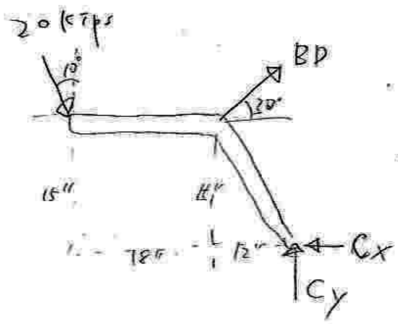
(1) Member ABC is supported by a pin connection at C and a link BD, which has a 4"x2" uniform cross section. The diameter of each pin is 3/8 in. Assume single shear at all the pin connections. If P=20 kips, find (a) the maximum average normal stress in link BD, (b) the average shearing stress in pin C, (c) the average shearing stress in pin B, and (d) the average bearing stress at B in link BD. (25%)

$P = 20$ kips





$$\circlearrowleft + \; \Sigma M_c = 0 \quad BD\cos 30^\circ(15) + BD\sin 30^\circ(12) + 20\sin 10^\circ(15) - 20\cos 10^\circ(30) = 0 \qquad \boxed{BD = 28.37 \text{ kips}} \; (T.)$$

$$\xrightarrow{+} \Sigma F_x = 0 \quad 20\sin 10^\circ + 28.37\cos 30^\circ - C_x = 0 \qquad \boxed{C_x = 28.04 \text{ kips}}$$

$$+\uparrow \Sigma F_y = 0 \quad -20\cos 10^\circ + 28.37\sin 30^\circ + C_y = 0 \qquad \boxed{C_y = 5.51 \text{ kips}}$$

$$C = \sqrt{C_x^2 + C_y^2} = \boxed{28.58 \text{ kips}}$$

(a) $$\sigma_{BD} = \frac{28.37}{(4-\frac{3}{8})(2)} = \boxed{3.913 \text{ ksi}}$$

(b) $$\tau_c = \frac{28.58}{\frac{1}{4}\pi(\frac{3}{8})^2} = \boxed{258.77 \text{ ksi}}$$

(c) $$\tau_B = \frac{28.37}{\frac{1}{4}\pi(\frac{3}{8})^2} = \boxed{256.9 \text{ ksi}}$$

(d) $$\sigma_b^{BD} = \frac{28.37}{(\frac{3}{8})(2)} = \boxed{37.83 \text{ ksi}}$$

Stresses at Interface

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(2) Two wooden members of 3.5 x 5.5-in uniform rectangular cross section are glued together along a plane interface forming and angle 20° with the horizontal. The ultimate normal stress of the wood is 9 ksi. At the glued interface, the ultimate shear stress is 240 psi and the ultimate normal stress is 480 psi. The design requires a factor of safety of 3.0. Determine the largest allowable axial load P that can be applied. (25%)

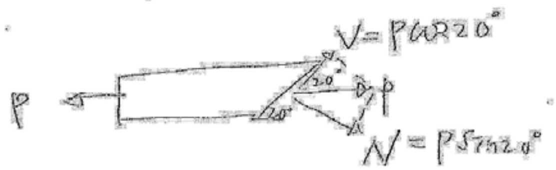
(a) Wood

$$\sigma_{allow} = \frac{P}{A_0}$$

$$\frac{9000}{3} = \frac{P}{(5.5)(3.5)}$$

$$\Rightarrow \boxed{P = 57.75 \text{ kips}}$$

(b) Interface



$$A = \frac{A_0}{\sin 20^\circ} = \frac{(5.5)(3.5)}{\sin 20^\circ} = 56.283 \text{ in}^2$$

normal $\sigma_{allow} = \frac{N}{A}$

$$\frac{480}{3} = \frac{P\sin 20^\circ}{56.283}$$

$$\Rightarrow \boxed{P = 26.33 \text{ kips}}$$

shear $\tau_{allow} = \frac{V}{A}$

$$\frac{240}{3} = \frac{P\cos 20^\circ}{56.283}$$

$$\Rightarrow \boxed{P = 4.79 \text{ kips}}$$

Pick the smallest $\boxed{P_{allow} = 4.79 \text{ kips}}$

Elongation

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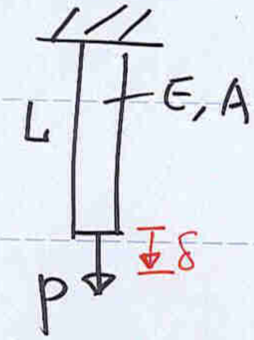
Elongation

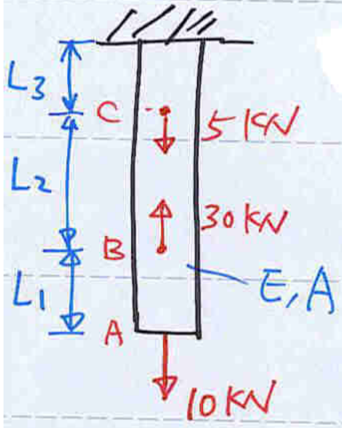


$$\sigma = E\varepsilon$$

$$\frac{P}{A} = E\frac{\delta}{L}$$

$$\Rightarrow \boxed{\delta = \frac{PL}{EA}}$$

<example> Find δ_A

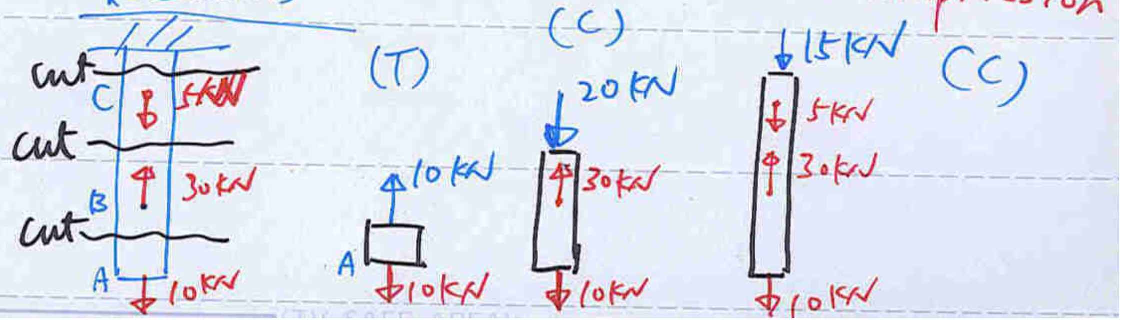


Note $\boxed{\delta_A = \sum \frac{PL}{EA}}$ → P has to be "internal force"

+ : tension

− : compression

Make a cut at different locations



Thermal Stresses

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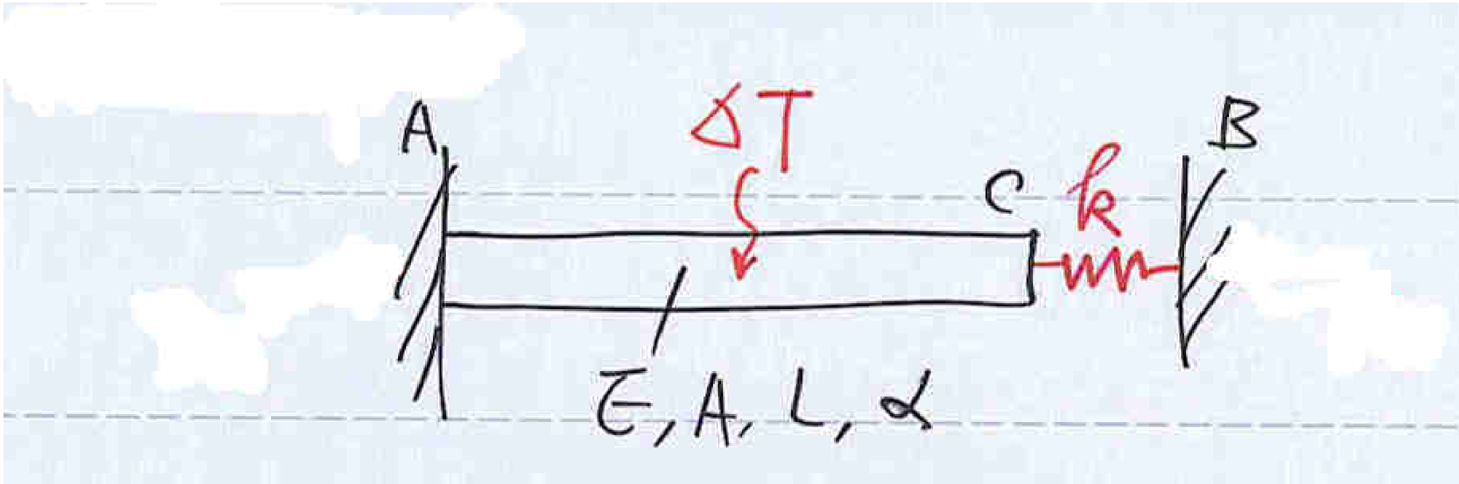
Find the normal stress in the bar due to temperature increase.



Composite Bar

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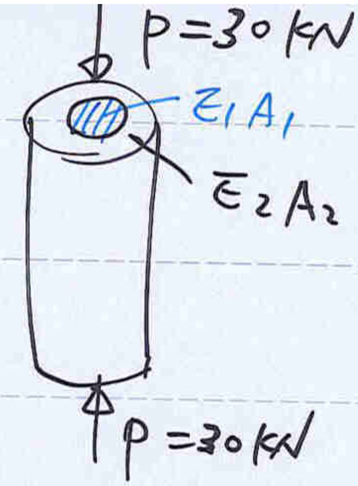
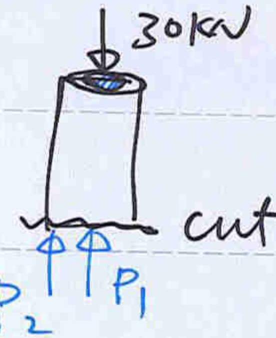
<Example>

Find
$\sigma_1, \sigma_2, \delta$

M#1 Long Way



FBD



EQ $\boxed{P_1 + P_2 = 30\ \text{kN}}$ — ①

Comp. $\delta_1 = \delta_2$

$$\boxed{\frac{P_1 L}{E_1 A_1} = \frac{P_2 L}{EA}} \quad \text{— ②}$$

Solve ① & ② for P_1 & P_2

$$\Rightarrow \sigma_1 = \frac{P_1}{A_1}, \quad \sigma_2 = \frac{P_2}{A_2}$$

$$\delta = \frac{P_1 L}{E_1 A_1} \text{ or } \frac{P_2 L}{E_2 A_2}$$

Torsion Shaft and Power Transmission

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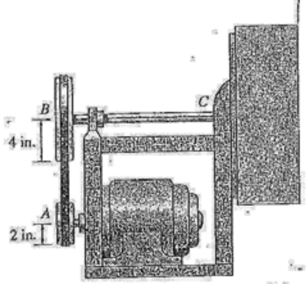
(3) The motor delivers 50 hp while turning at a constant rate of 1350 rpm at A. Using the belt and pulley system this torque is delivered to the steel blower shaft BC. Determine the smallest diameter of the shaft if the allowable shear stress for the steel is 12 ksi. (25%)



$P = 50\ hp = 50\ (550)(12)$
$= 330{,}000\ in\text{-}lb/s$

$\omega_A = 1350\ rpm = \frac{1350(2\pi)}{60}$
$= 141.37\ rad/s$

$P = T_A \omega_A$, $T_A = \frac{P}{\omega} = \frac{330000}{141.37} = 2334.3\ lb\text{-}in$

$\frac{T_A}{r_A} = \frac{T_B}{r_B}$ $T_B = T_A\left(\frac{r_B}{r_A}\right) = 2334.3\left(\frac{4}{2}\right) = 4668.6\ lb\text{-}in$

$\tau_{allow} = \frac{Tc}{J}$ $J = \frac{\pi}{2}c^4$

$12\times10^3 = \frac{4668.6\ c}{\frac{\pi}{2}c^4} = \frac{2(4668.6)}{\pi\ c^3}$

$\boxed{c = 0.628\ in}$

$\boxed{d = 1.256\ in}$

Bending Stresses

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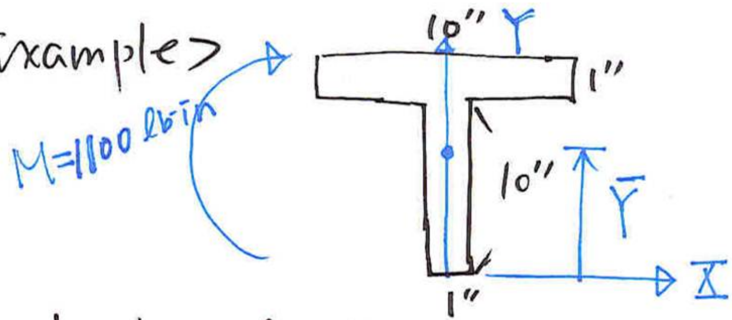
<Example>



Find $\sigma_{max}^{Tension}$ & $\sigma_{max}^{Compression}$

Step 1 Locate C

	A_i	Y_i
	10(1) =10	10.5
	1(10) =10	5

$$\bar{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} \quad (GPA)$$

$$= \frac{10(10.5)+10(5)}{10+10} = 7.75''$$

Step 2 Find I

$$I = \frac{1}{12}(1)(10)^3 + 10(1)(7.75-5)^2 + \frac{1}{12}(10)(1)^3 + 10(1)(10.5-7.75)^2 = 235.417 \text{ in}^4$$

Step 3

$$\sigma_{bottom} = \frac{1100(7.75)}{235.417} = 36.2 \text{ psi}$$ Maximum tensile stress

$$\sigma_{Top} = -\frac{1100(11-7.75)}{235.417} = -15.19 \text{ psi}$$ Maximum Compressive stress

Eccentric Loading

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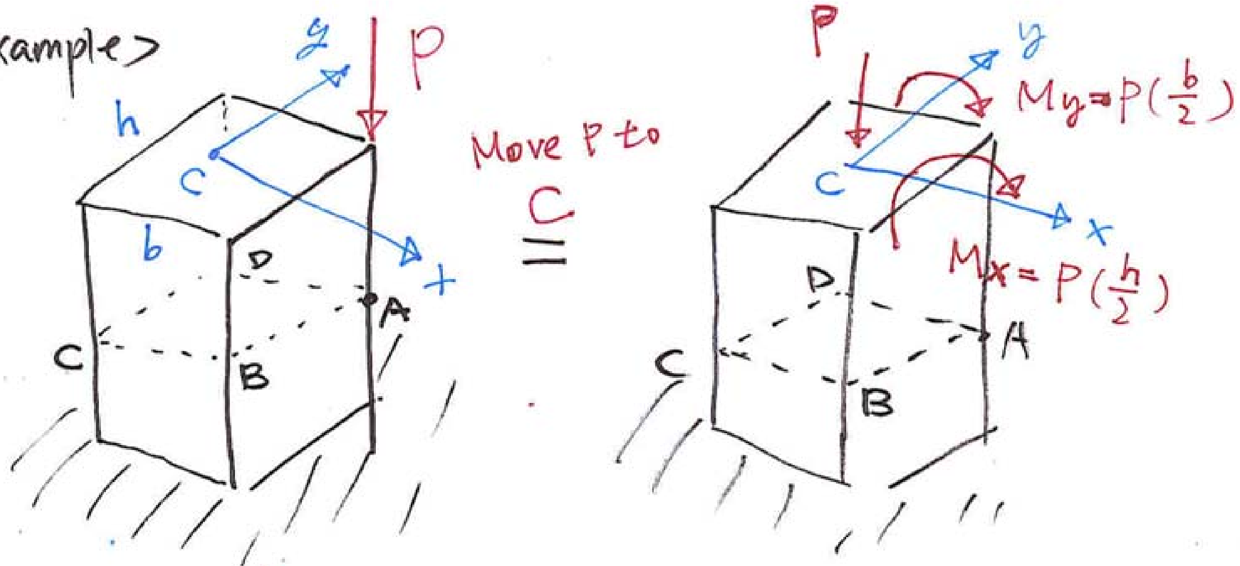
<Example>



$$\begin{cases} M_x = P(\frac{h}{2}) & \text{moment about the } x \text{ axis} \\ M_y = P(\frac{b}{2}) & \text{moment about the } y \text{ axis} \end{cases}$$

$I_x = \frac{1}{12} b h^3$

$I_y = \frac{1}{12} b^3 h$

$$\begin{cases} \sigma_A = -\frac{P}{A} - \frac{M_x(\frac{h}{2})}{I_x} - \frac{M_y(\frac{b}{2})}{I_y} \\ \sigma_B = -\frac{P}{A} + \frac{M_x(\frac{h}{2})}{I_x} - \frac{M_y(\frac{b}{2})}{I_y} \\ \sigma_C = -\frac{P}{A} + \frac{M_x(\frac{h}{2})}{I_x} + \frac{M_y(\frac{b}{2})}{I_y} \\ \sigma_D = ? \; \cdots \end{cases}$$

In general

$$\sigma = -\frac{P}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y}$$

Transverse Shear

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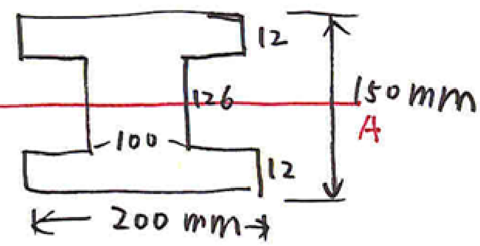
<example>



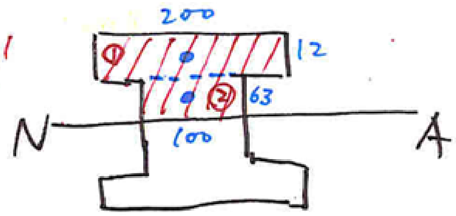
Find τ_{max} if $V = 10\ kN$

most likely at N.A. for this problem

Step 1 Locate N.A.

Step 2 $I = \frac{1}{12}(200)(150)^3 - 2\left[\frac{1}{12}(50)(126)^3\right] = 39580200\ mm^4$
$= 3.95802 \times 10^{-5}\ m^4$

Step 3 Shade A'



$$Q = \Sigma A_i \bar{y}_i = \underbrace{(200)(12)}_{A_1}\underbrace{\left(63+\frac{12}{2}\right)}_{\bar{y}_1} + \underbrace{(100)(63)}_{A_2}\underbrace{\left(\frac{63}{2}\right)}_{\bar{y}_2} = 364050\ mm^3$$

$= 3.6405 \times 10^{+5}\ mm^3$
$= 3.6405 \times 10^{-4}\ m^3$

$t = 100\ mm = 0.1\ m$
(where you cut to define A')

$$\tau_{max} = \frac{VQ}{It} = \frac{(10\times10^3)(3.6405\times10^{-4})}{(3.95802\times10^{-5})(0.1)} = \boxed{0.92\ MPa}$$

Built-Up Beams

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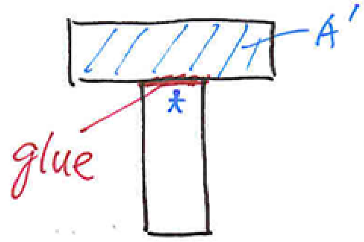
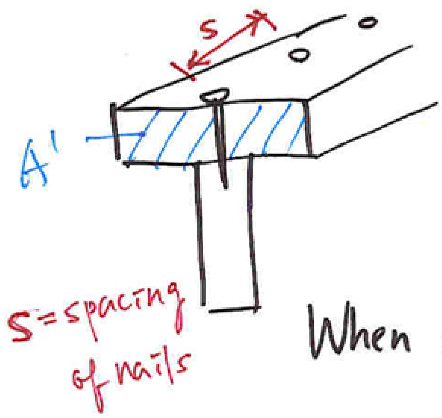
Shear Flow in Built-Up Beams



$$\tau = \frac{VQ}{It}$$

When designing, Compare τ to τ_{allow}^{glue}



τ has no meaning at the A' interface

Define $\boxed{\text{shear flow}}$ $q = \tau t = \frac{VQ}{I}$

= shear force per unit length

When designing, do

$$q = \underbrace{\frac{VQ}{I} = \frac{F_{nail}}{s}}_{\text{Force per unit length}}$$

Allowable nail force. Eeach nail covers a distance of s along the beam

Where $F_{nail} = \left(\frac{1}{4}\pi d^2\right) \tau_{allow}^{nail}$

Thin-Walled Vessels

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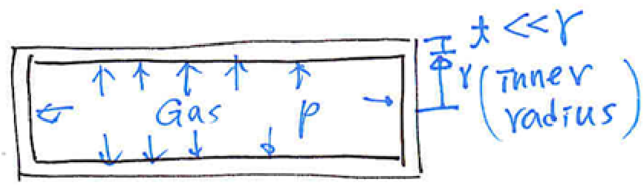
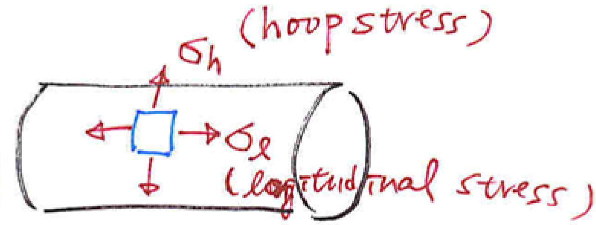
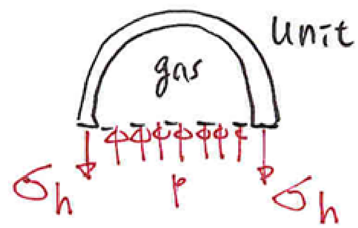
Thin-Walled Pressure Vessels

I. Cylindrical Vessels





hoop stress

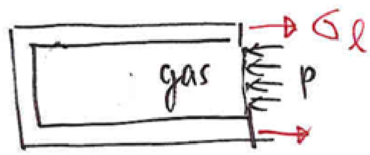


$+\uparrow \Sigma F_y = 0$

$$p(2r)\cancel{(1)} - 2\sigma_h(t)\cancel{(1)} = 0$$

$$\boxed{\sigma_h = \frac{pr}{t}}$$

longitudinal stress



$\overset{+}{\rightarrow} \Sigma F_x = 0$

$$\sigma_\ell\, t(2\pi r) - p(\pi r^2) = 0$$

$$\boxed{\sigma_\ell = \frac{pr}{2t}}$$

Strain Gauge

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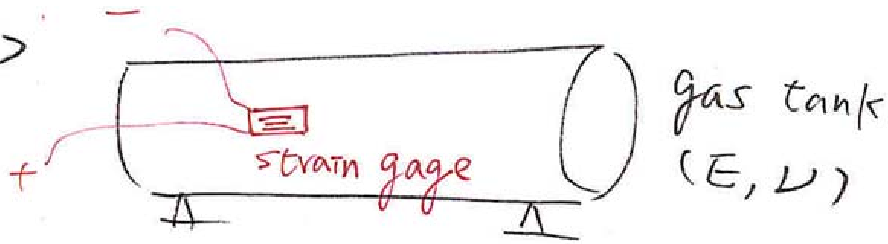
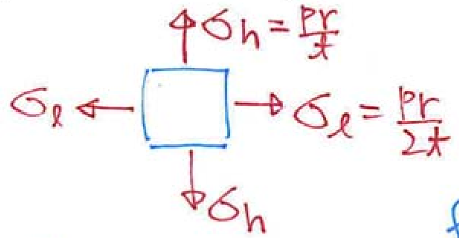
<Example>



If ε_x is measured by the attached strain gage (before the gas is filled), find the gas pressure p.

Sol:



free exterior surface

$$\boxed{\varepsilon_x} = \frac{1}{E}\left(\sigma_x - \nu\sigma_y - \nu\cancel{\sigma_z}^{\,0}\right) = \boxed{\frac{1}{E}\left(\frac{pr}{2t} - \nu\frac{pr}{t}\right)}$$

from strain gage.

$\Rightarrow$ solve for p.

$$p = \frac{E\varepsilon_x t}{r\left(\frac{1}{2} - \nu\right)}$$

Bulking-Rigid Column

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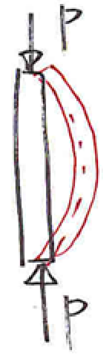
Buckling of Columns

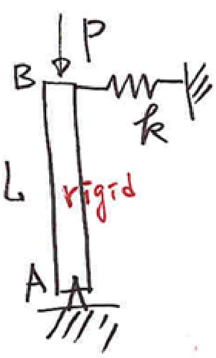


When P reaches P_{cr}, the columns may "buckle".

Stability

Part I Rigid Columns

<example>

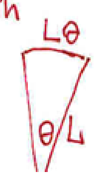


Find P_{cr}

"Invite this to a party".

$\boxed{\begin{array}{l}\text{Small } \theta \text{ (radians)} \\ \sin\theta \approx \theta \\ \cos\theta \approx 1\end{array}}$

or Arc length



FBD

$\circlearrowleft + \sum M_A = 0$ → Equilibrium at θ

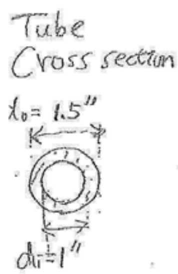
$$P\underbrace{(L\cancel{\theta})}_{\text{moment arm}} - kL\cancel{\theta}\underbrace{(L)}_{\text{moment arm}} = 0$$

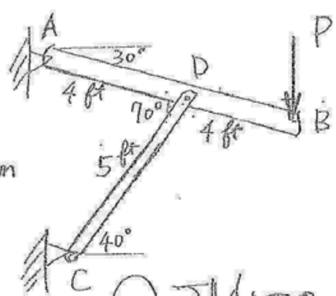
$$\Rightarrow \boxed{P_{cr} = kL}$$

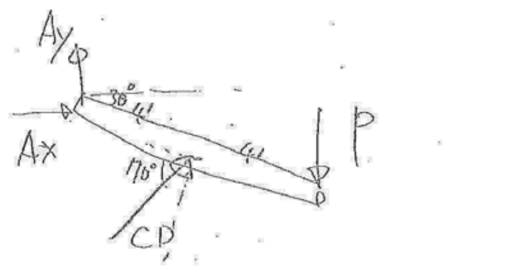
Bulking-Elastic Column

Wednesday, February 16, 2011
4:41 PM

(1) Determine the greatest load P the frame will support without causing the steel link CD ($E = 29\times10^3$ ksi, Yield strength $\sigma_Y = 36$ ksi) to yield or to buckle. Use a factor of safety of 2.0. Link CD has a tube cross section as shown. (25%)







$\sum M_A = 0$ $\quad P(8\cos 30°) - CD\sin 70° (4) = 0$

$\boxed{CD = 1.8432P}$ Compression

To yield $\quad \sigma_Y = 36 \text{ ksi} = \dfrac{1.8432P}{\frac{1}{4}\pi(1.5^2 - 1^2)}$

$\boxed{P = 19.175 \text{ kips}}$

To buckle $\quad CD_{cr} = \dfrac{\pi^2 EI}{L^2}$ $\qquad I = \dfrac{\pi}{4}\left[\left(\dfrac{1.5}{2}\right)^4 - (0.5)^4\right] = 0.19942 \text{ in}^4$

$L = 5(12)$ in

$1.8432P = \dfrac{\pi^2(29\times10^3)(0.19942)}{[5(12)]^2}$

$\boxed{P = 8.6018 \text{ kips}}$

Pick the smaller P $\qquad P_{allow} = \dfrac{8.6018}{F.S.} = \dfrac{8.6018}{2} = \boxed{4.30 \text{ kip}}$