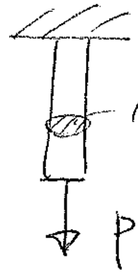
FE Review – Solids

Stresses

Normal stress

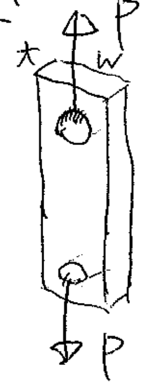


$$\sigma = \frac{P}{A}$$

$$\begin{Bmatrix} Pa \\ psi \end{Bmatrix} = \begin{Bmatrix} N/m^2 \\ lb/in^2 \end{Bmatrix}$$

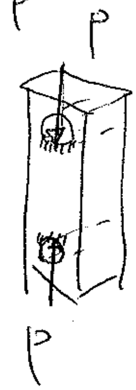
Pin-hole effect

Tension



$$\sigma = \frac{P}{(W-d)t}$$

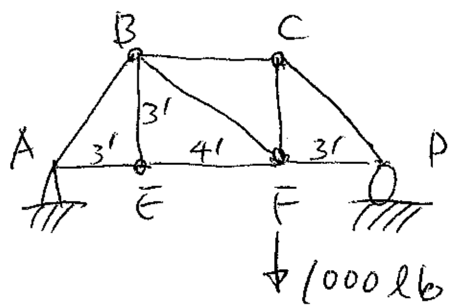
Compression



$$\sigma = \frac{P}{Wt}$$

pin hole has no effect

<Example>



Find σ_{BC}, σ_{BF}, σ_{EF}

Step1 Find A_y, D_y

Step2 Method of section

Step3 Stress

However, if compression, consider bulking

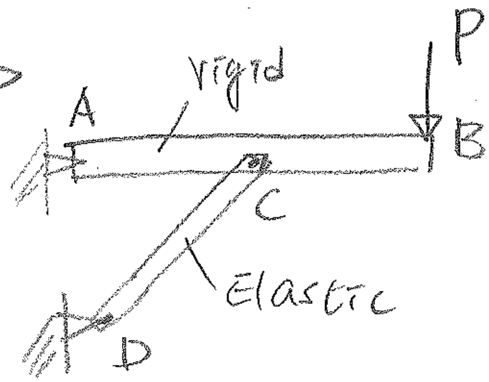
$$P_{cr} = \frac{\pi^2 EI}{(kL)^2}$$

- pin-pin $k=1$
- pin-fixed $k=0.7$
- fixed-fixed $k=0.5$
- free-fixed $k=2$

Which one is most stable?

Which one is the least?

<Example>

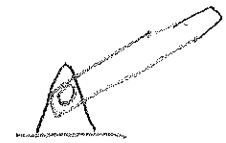


Given: dimensions
E, I σ_{allow}

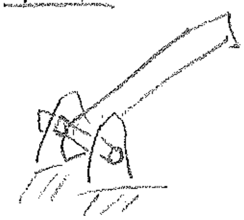
Find P_{allow}

<u>shear stress</u>

Single shear $\tau = \dfrac{V}{A}$



Double shear $\tau = \dfrac{V}{2A}$

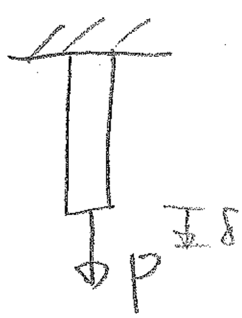


<Example> Find τ at D in the above example.

<u>Step 1</u> Find F_{CD}

<u>Step 2</u> Single shear $\tau = \dfrac{F_{CD}}{\frac{1}{4}\pi d^2}$ d = pin dia

<u>Elongation</u>



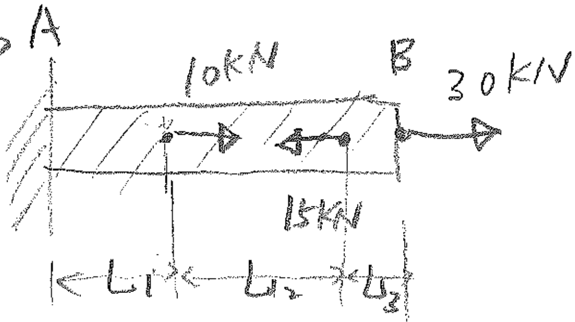
$$\delta = \frac{PL}{EA}$$

Why? $\sigma = E\varepsilon$

$$\frac{P}{A} = E\frac{\delta}{L}$$

$$\Rightarrow \delta = \frac{PL}{EA}$$

<Example>

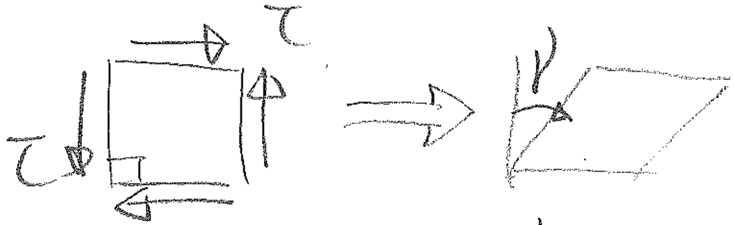


Find δ_B

$$\delta_B = \Sigma \frac{PL}{EA}$$

P → Internal

Shear strain

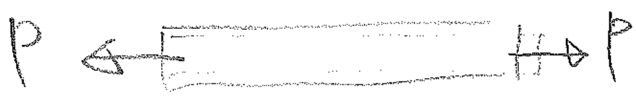


γ in radian

$$\tau = G\gamma$$

$$G = \frac{E}{2(1+\nu)}$$

Poisson's Ratio ν



$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{P}{EA}$$

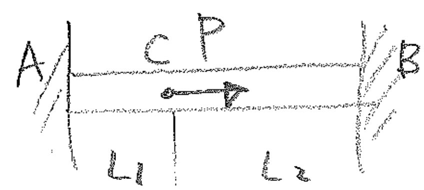
$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x$$

Volumetric strain $= \frac{\Delta V}{V} = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z) - 1$

$$\approx \varepsilon_x + \varepsilon_y + \varepsilon_z$$

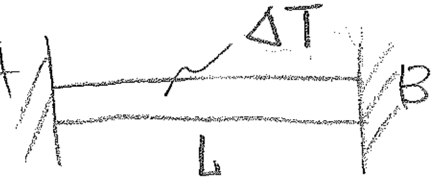
Statically Indeterminate Bars

<Example>



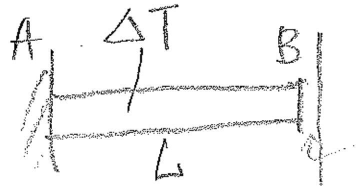
Find R_A, R_B, σ_{AC}, σ_{BC}

<Example>



Find σ_{AB} when ΔT occurs

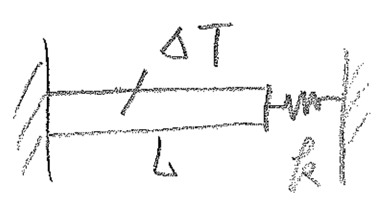
<Example>



spacing $= d$

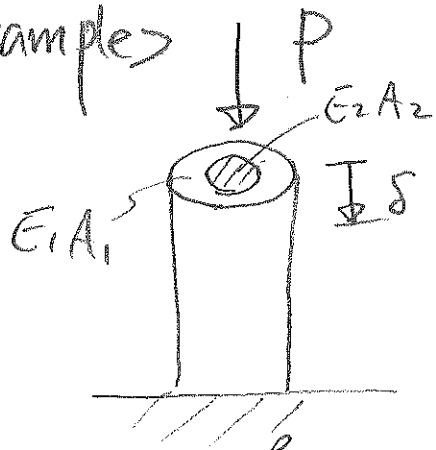
Find σ_{AB} when ΔT occurs

<Example>



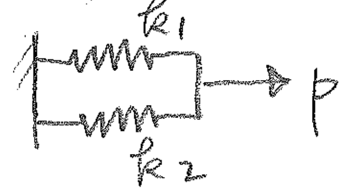
Find σ_{AB} when ΔT occurs

<example>

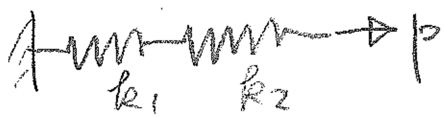


Find δ

Note

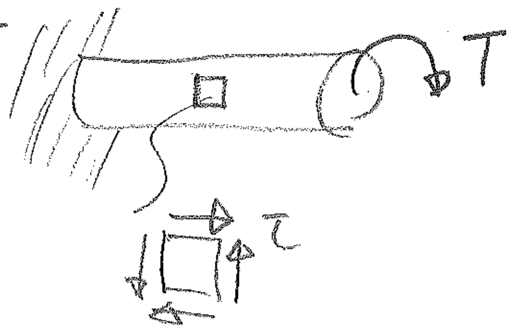


$k_{eq} = ?$



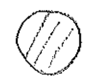
$k_{eq} = ?$

Torsion



$\tau_{max} = \frac{Tc}{J}$

$J = \int \rho^2 dA = \begin{cases} \frac{\pi}{2} c^4 \\ \frac{\pi}{2}(c_o^4 - c_i^4) \end{cases}$



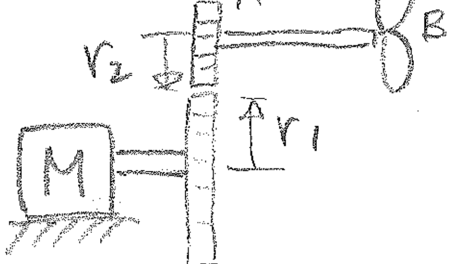
Angle of twist

$\phi = \frac{TL}{GJ}$

Power Transmission

$\mathbb{P} = T\omega$

1 hp = 550 ft-lb/s = 746 W



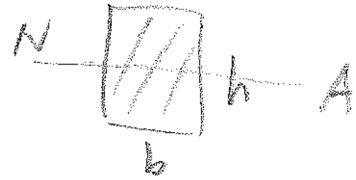
Gears

Given: motor power $\mathbb{P}$ @ 3000 rpm

Find τ_{max} in shaft AB

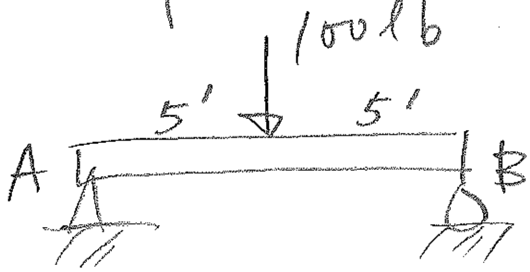
Bending

$$\sigma = \frac{My}{I}$$

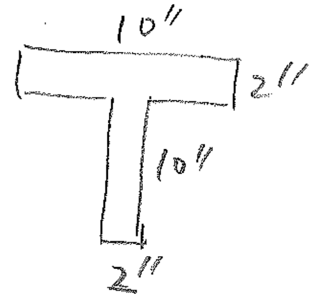


$$I = \frac{1}{12} b h^3$$

<Example>

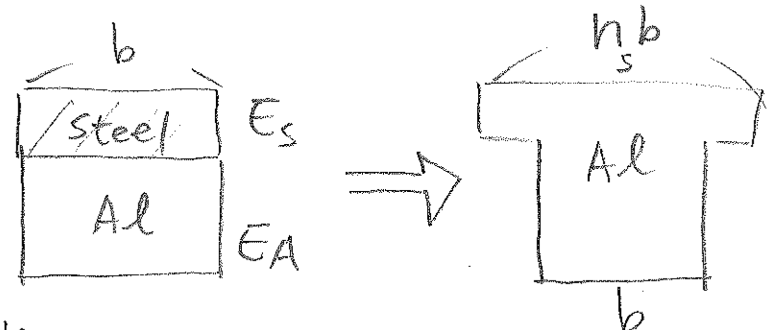


Cross section



Find σ_{max}

Composite Beams

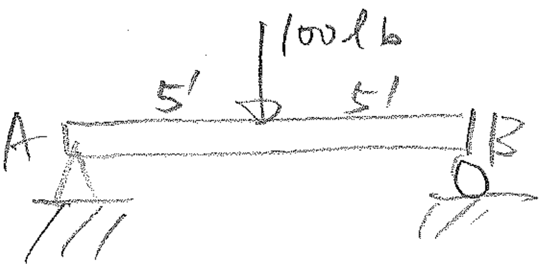


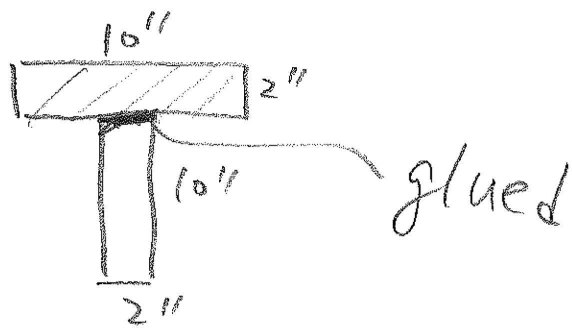
$$n_s = \frac{E_s}{E_A}$$

$$n_A = 1$$

$$\sigma = n \frac{My}{I}$$

Transrese shear

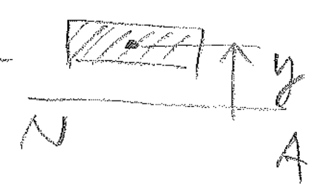




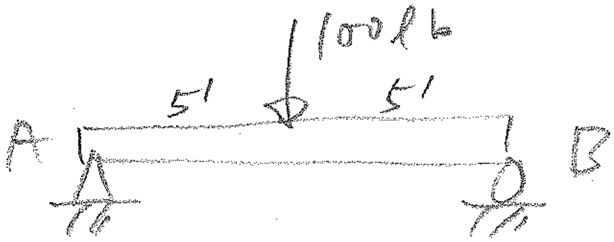
$$\tau = \frac{VQ}{It}$$

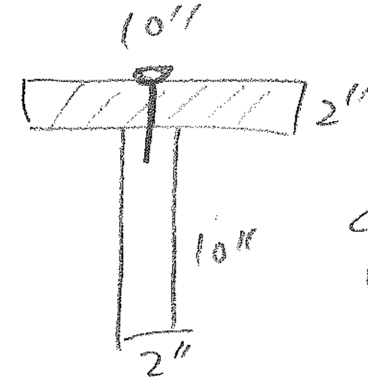
$t = 2''$

$Q = \Sigma A_i \bar{y}_i$ of



Shear Flow





$$q = \tau t = \frac{VQ}{I}$$

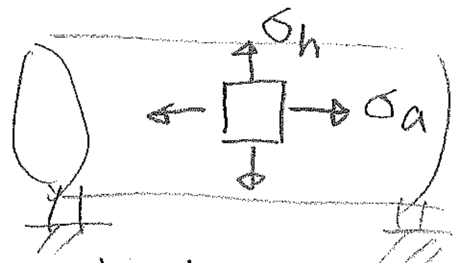
= shear force per unit length along the beam

Then $\frac{F_{nail}}{s} = q$

s ← nail spacing

Thin-Walled Pressure Vessels

Cylindrical



$$\sigma_h = \frac{Pr}{t}$$

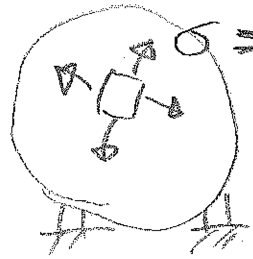
t = wall thickness

$$\sigma_a = \frac{Pr}{2t}$$

h: hoop

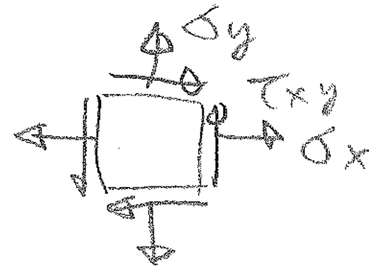
a: axial

Spherical



$$\sigma = \frac{Pr}{2t}$$

Multi-axial Loading



$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

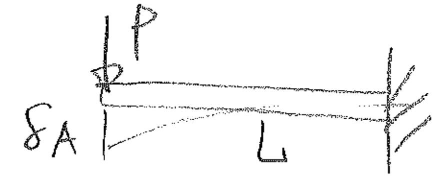
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x - \nu\sigma_z)$$

$$\gamma_{xy} = \tau_{xy}/G$$

Deflection of Beams

$$EI\,y'' = M(x)$$

<Example>



$\delta_A = ?$

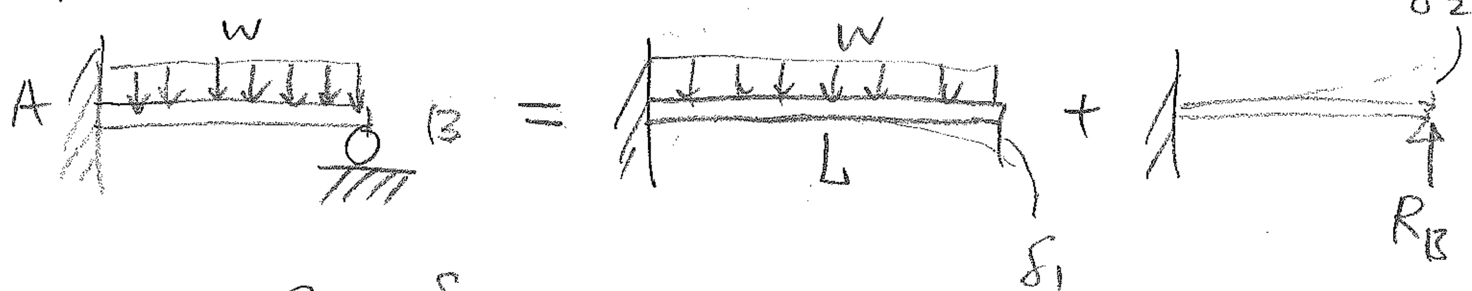
$$EI\,y'' = -Px$$
$$EI\,y' = -P\frac{x^2}{2} + C_1$$
$$EI\,y = -P\frac{x^3}{6} + C_1 x + C_2$$

B.C's: $x = L \quad y = 0$

$x = L \quad y' = 0$

$\Rightarrow C_1 = ? \quad C_2 = ?$

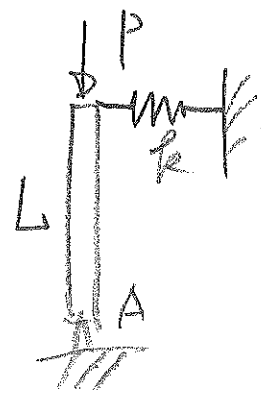
Superposition



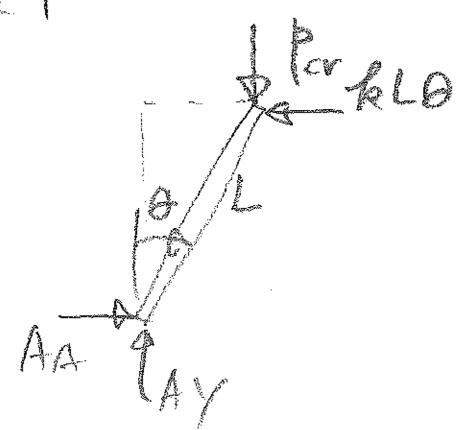
$$\delta_1 = \delta_2$$

$\Rightarrow$ Table $\quad \dfrac{wL^4}{8EI} = \dfrac{R_B L^3}{3EI} \quad \Rightarrow R_B = ?$

Bulking



Find P_{cr}



$\circlearrowleft + \; \Sigma M_A = 0$

$$P_{cr}(L\theta) - kL\theta(L) = 0$$

$$P_{cr} = kL$$

small θ